MULTI-MODE FREQUENCY-FILTERED CORRELATIONS

A NOVEL METHOD FOR CALCULATING FREQUENCY-FILTERED PHOTON CORRELATIONS

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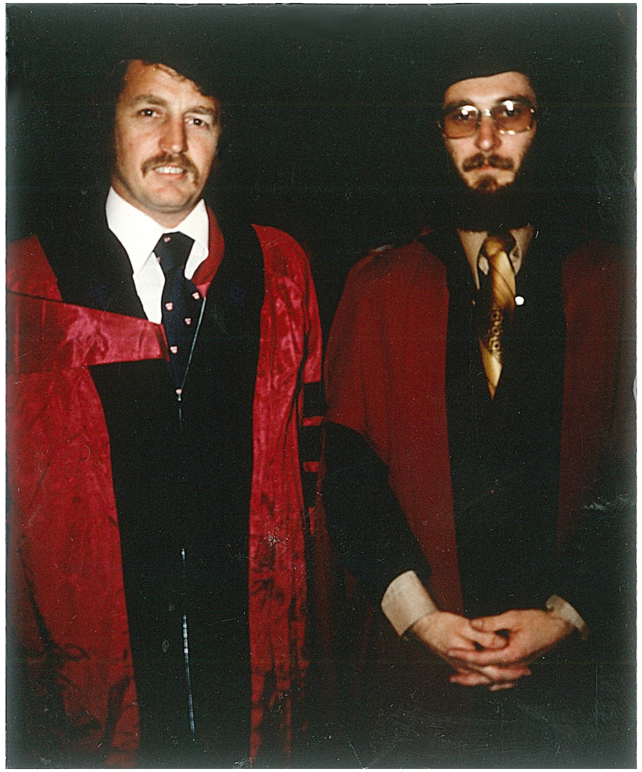
ANTIBUNCHING – A QUANTUM EFFECT



Proposal for the measurement of the resonant Stark effect by photon correlation techniques

H J Carmichael† and D F Walls
School of Science, University of Waikato, Hamilton, New Zealand

Received 8 December 1975

- Second-order correlation function:

$$g^{(2)}(\tau) = \frac{\langle \sigma_+(0)\sigma_+\sigma_-(\tau)\sigma_-(0)\rangle_{ss}}{\langle \sigma_+\sigma_-\rangle_{ss}^2}$$

- $g^{(2)}(0) = 0$: antibunching / single-photons
- $g^{(2)}(0) = 1$: uncorrelated / random
- $g^{(2)}(0) > 1$: bunching / multi-photons

J. Phys. B: At. Mol. Phys. 9, L43 (1976)

THREE-LEVEL LADDER-TYPE ATOM

- Hamiltonian and master equation

$$H_A = -\hbar\left(\frac{\alpha}{2} + \delta\right)|e\rangle\langle e| - 2\hbar\delta|f\rangle\langle f| + \hbar\frac{\Omega}{2}\left(\Sigma_+ + \Sigma_-\right)$$

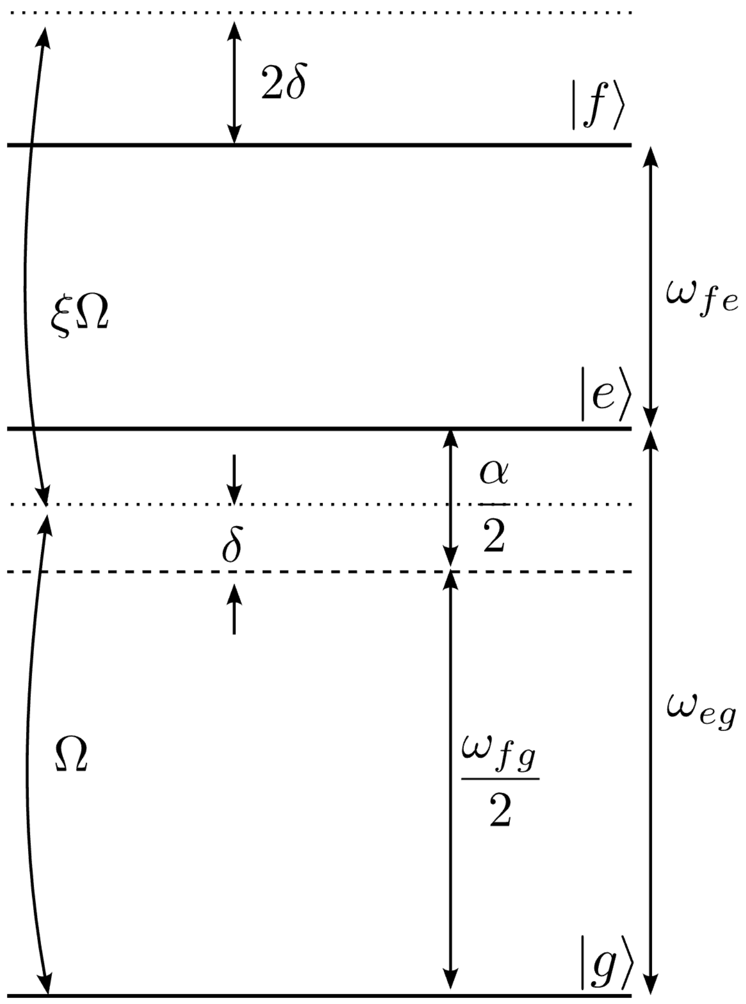
$$\frac{\mathrm{d}\rho}{\mathrm{d}t} = \frac{1}{i\hbar}[H_A, \rho] + \frac{\Gamma}{2}\Lambda(\Sigma_-)\rho$$

- Atomic lowering operator

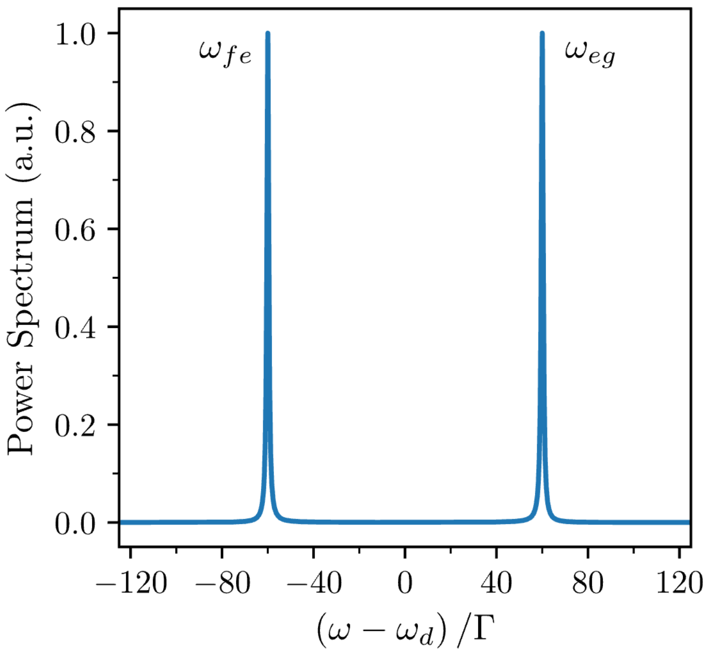
$$\Sigma_- = |g\rangle\langle e| + \xi|e\rangle\langle f|$$

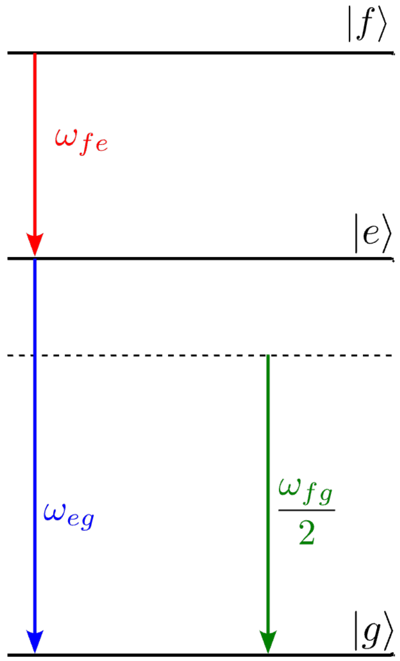
- α – anharmonicity
- δ - drive detuning from two-photon resonance
- ξ - dipole moment ratio
- Γ - atomic decay rate

$\Lambda(X)\bullet = 2X \bullet X^\dagger - X^\dagger X \bullet - \bullet X^\dagger X$

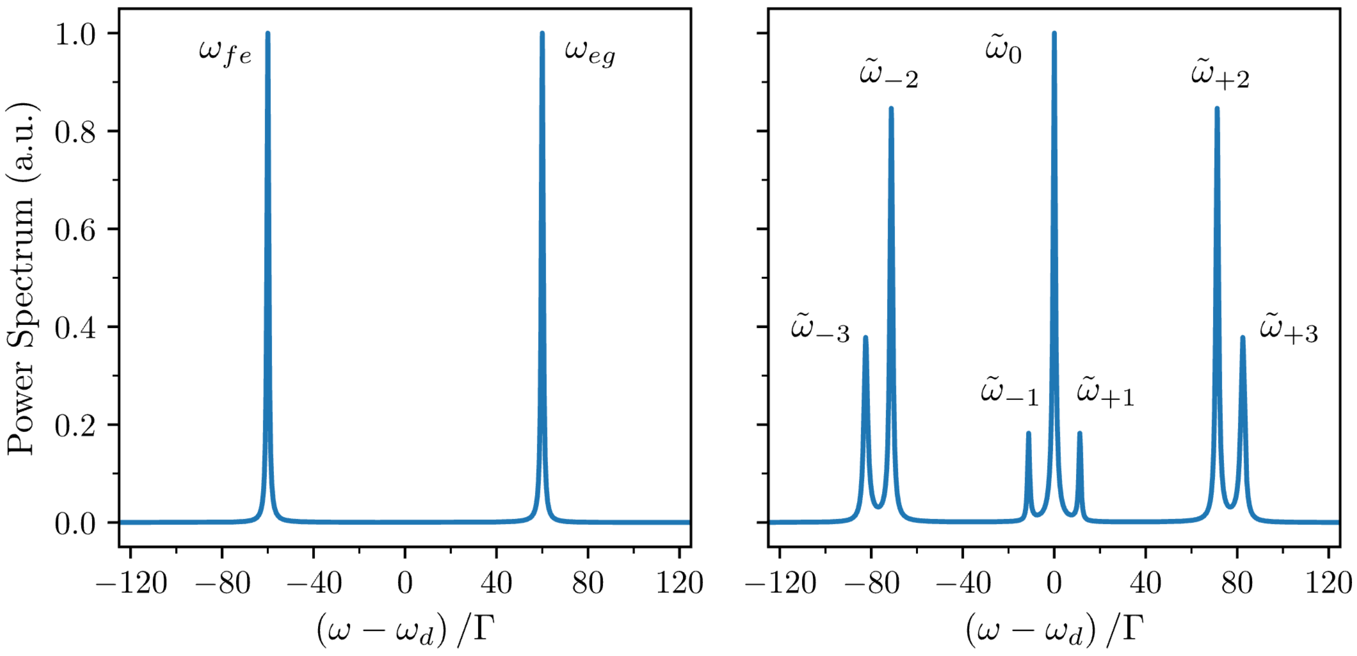


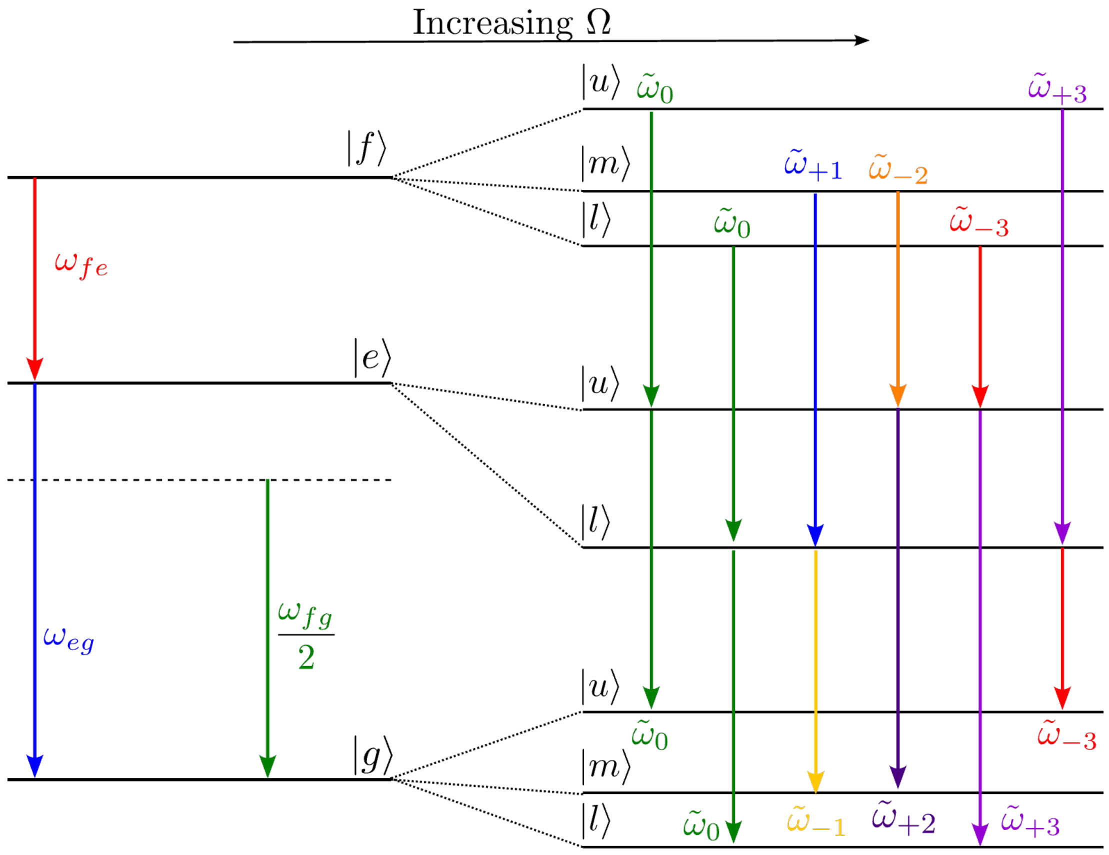
THREE-LEVEL LADDER-TYPE ATOM





THREE-LEVEL LADDER-TYPE ATOM





FREQUENCY FILTERING

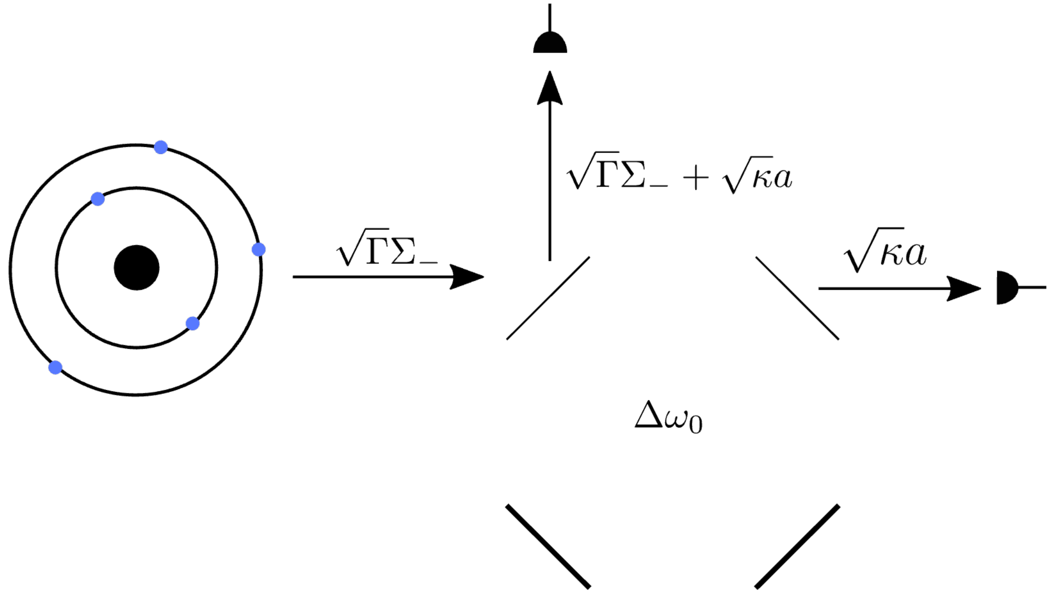
Detector atom approach – PRL **109**, 183601 (2012).

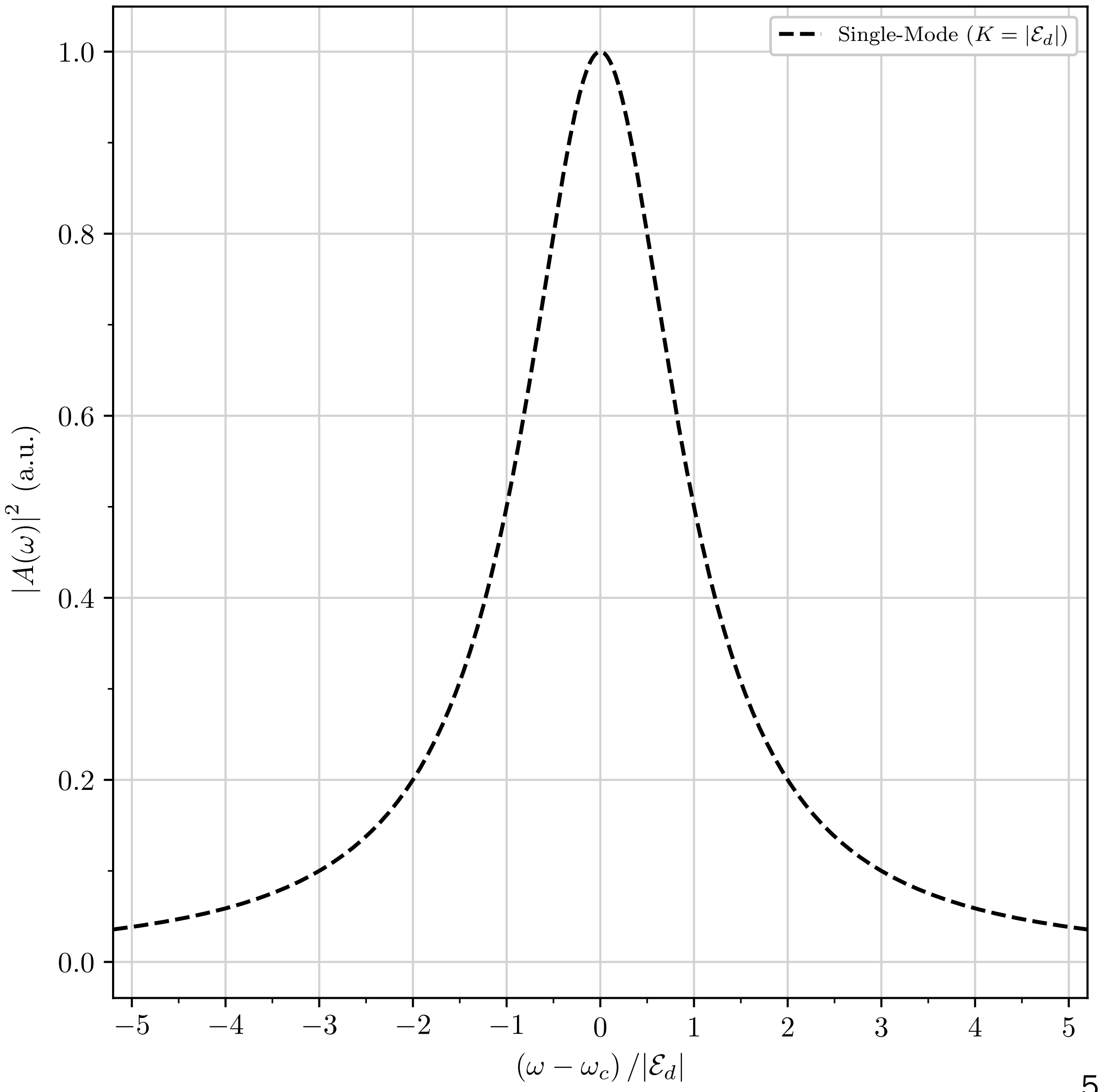
PRL **109,** 183601 (2012) PHYSICAL REVIEW LETTERS week ending 2 NOVEMBER 2012

Theory of Frequency-Filtered and Time-Resolved *N*-Photon Correlations

E. del Valle,[1,*] A. Gonzalez-Tudela,[2] F. P. Laussy,[2,3] C. Tejedor,[2] and M. J. Hartmann[1]

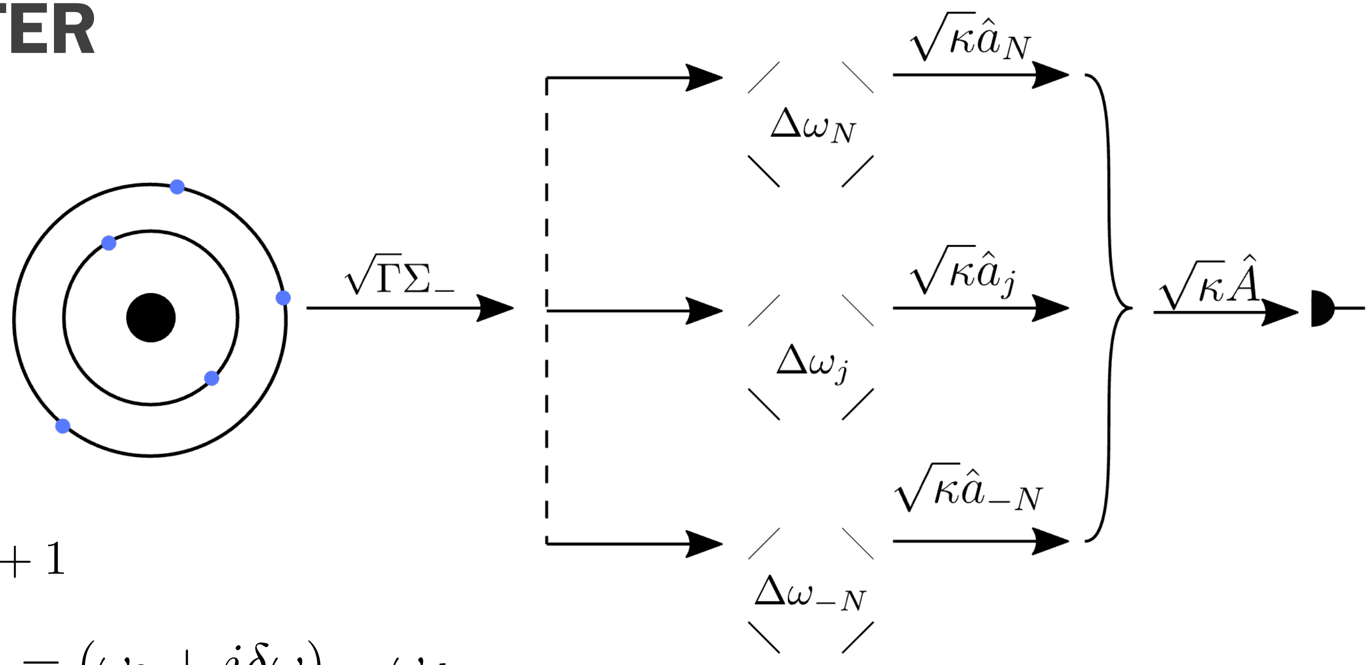
Single-mode cavity filters





THE MULTI-MODE ARRAY FILTER

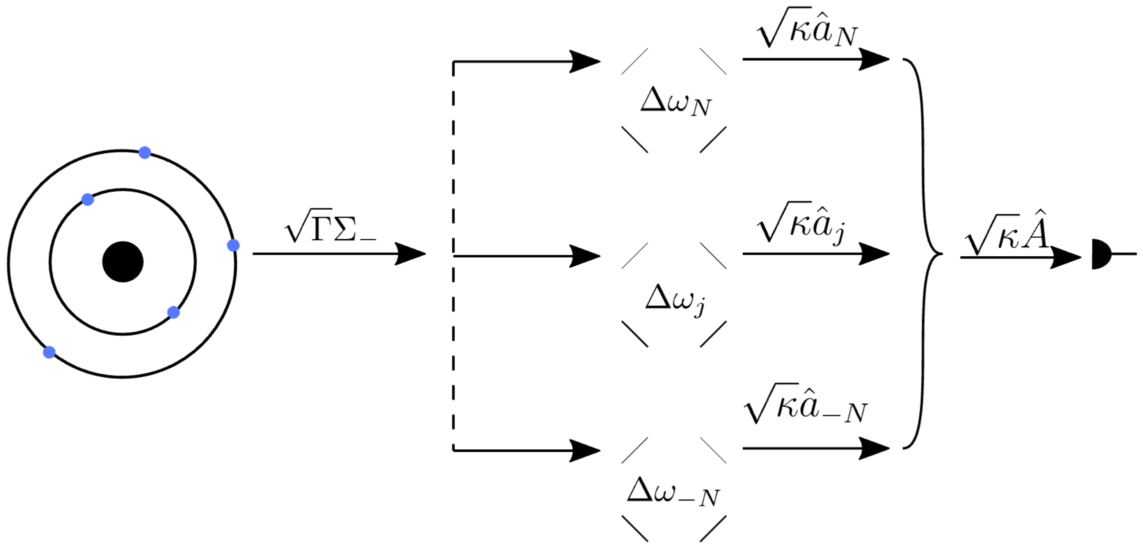
$$H_F = \hbar \sum_{j=-N}^{N} \Delta\omega_j a_j^\dagger a_j$$

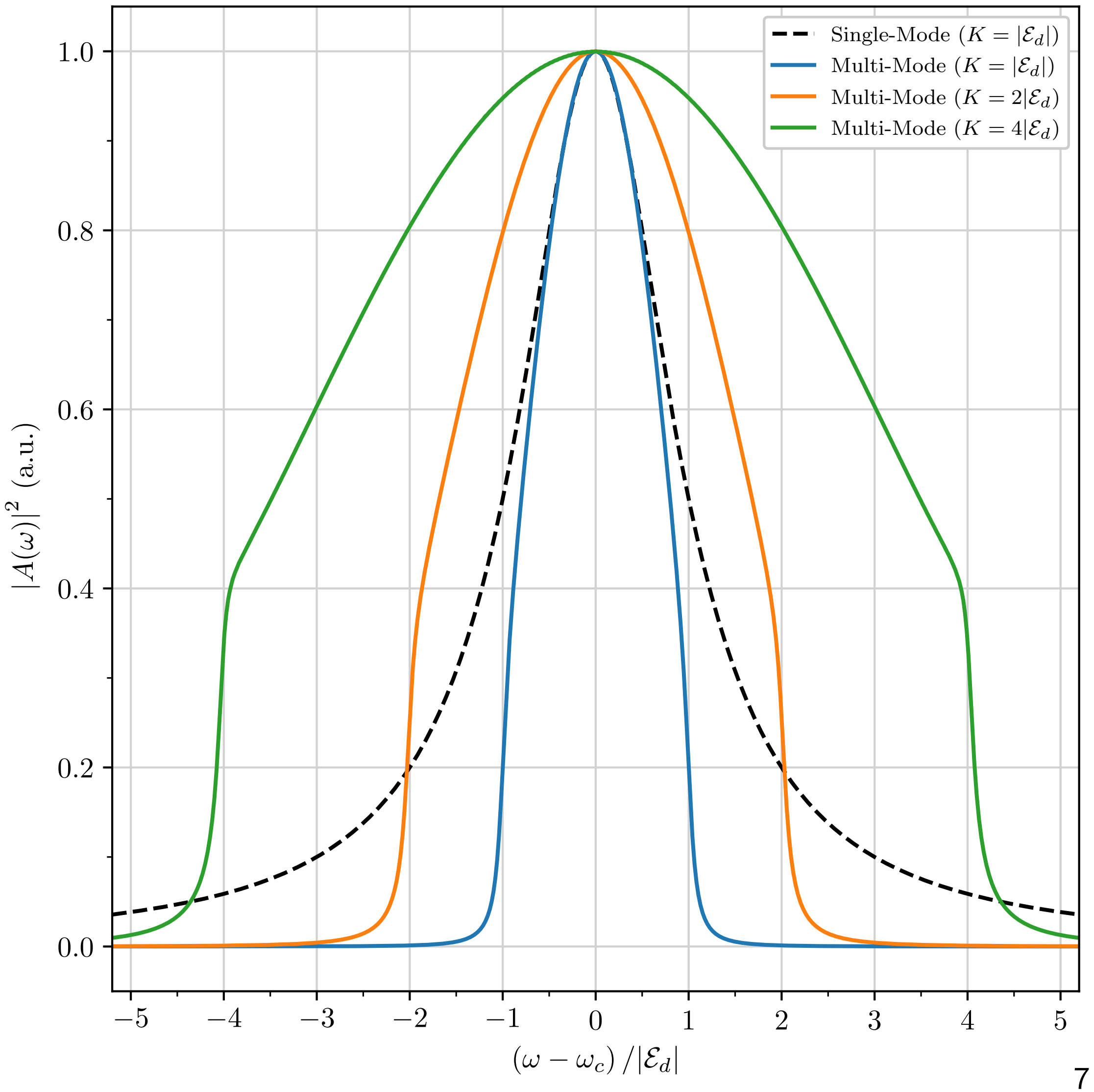


- Total number of filter modes – $2N+1$
- Mode detuning from drive frequency – $\Delta\omega_j = (\omega_0 + j\delta\omega) - \omega_d$
- Frequency separation – $\delta\omega$
- Mode-dependent phase modulation
- Combined/collective output

THE MULTI-MODE ARRAY FILTER

- Cascade fluorescence into an *array* of single-mode cavities
- Sharper frequency response cut-off
- Larger bandwidth = faster (better) temporal response
- Multi-mode "halfwidth" $(N\delta\omega)$, Single-mode halfwidth (κ)

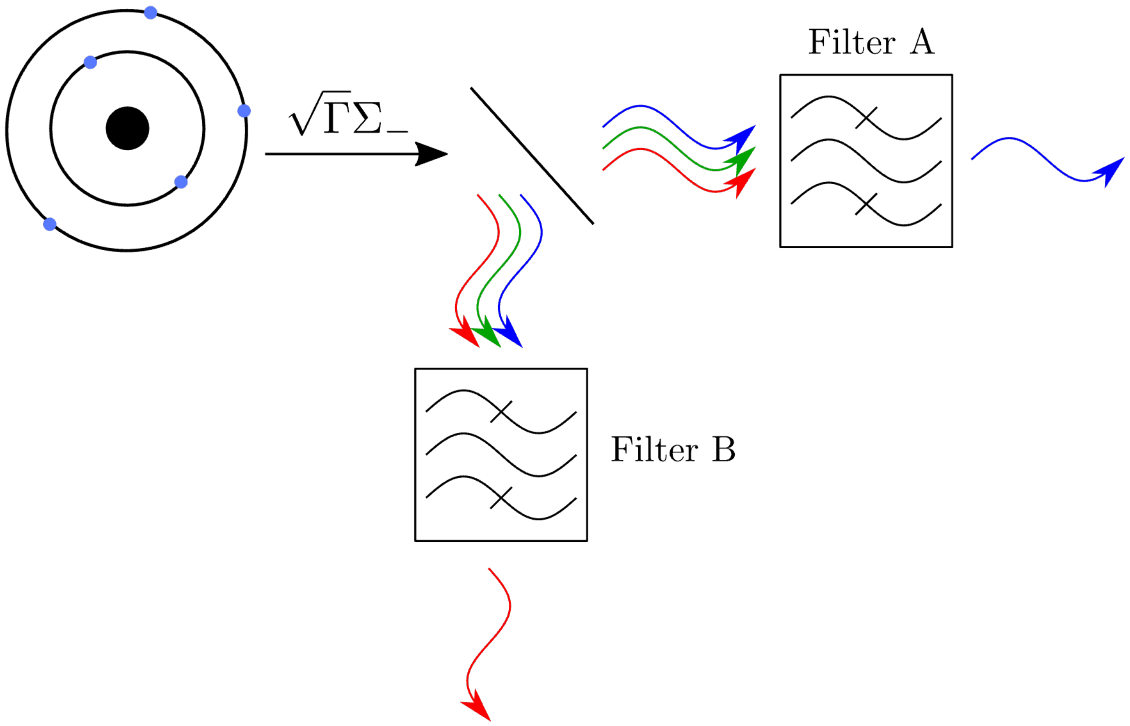




THE MULTI-MODE ARRAY FILTER

Master equation

$$\begin{aligned}\frac{\mathrm{d}\rho}{\mathrm{d}t} = \frac{1}{i\hbar}[H_A, \rho] + \frac{\Gamma}{2}\Lambda(\Sigma_-)\rho \\ - i \sum_{j=-N}^{N} \Delta\omega_j^{(a)} a_j^\dagger a_j + \frac{\kappa}{2} \sum_{j=-N}^{N} \Lambda(a_j)\rho \\ - \sum_{j=-N}^{N} \mathcal{E}_j \left(a_j^\dagger \Sigma_- \rho - \Sigma_- \rho a_j^\dagger \right) - \sum_{j=-N}^{N} \mathcal{E}_j^* \left(\rho \Sigma_+ a_j - a_j \rho \Sigma_+ \right) \\ - i \sum_{j=-N}^{N} \Delta\omega_j^{(b)} b_j^\dagger b_j + \frac{\kappa}{2} \sum_{j=-N}^{N} \Lambda(b_j)\rho \\ - \sum_{j=-N}^{N} \mathcal{E}_j \left(b_j^\dagger \Sigma_- \rho - \Sigma_- \rho b_j^\dagger \right) - \sum_{j=-N}^{N} \mathcal{E}_j^* \left(\rho \Sigma_+ b_j - b_j \rho \Sigma_+ \right)\end{aligned}$$



$$\Lambda(X)\bullet = 2X \bullet X^\dagger - X^\dagger X \bullet - \bullet X^\dagger X$$

THE MULTI-MODE ARRAY FILTER

Master equation

$$\frac{\mathrm{d}\rho}{\mathrm{d}t} = \frac{1}{i\hbar}[H_A, \rho] + \frac{\Gamma}{2}\Lambda(\Sigma_-)\rho$$ Driven atom

$$- i \sum_{j=-N}^{N} \Delta\omega_j^{(a)} a_j^\dagger a_j + \frac{\kappa}{2} \sum_{j=-N}^{N} \Lambda(a_j)\rho$$

$$- \sum_{j=-N}^{N} \mathcal{E}_j \left(a_j^\dagger \Sigma_- \rho - \Sigma_- \rho a_j^\dagger\right) - \sum_{j=-N}^{N} \mathcal{E}_j^* \left(\rho \Sigma_+ a_j - a_j \rho \Sigma_+\right)$$

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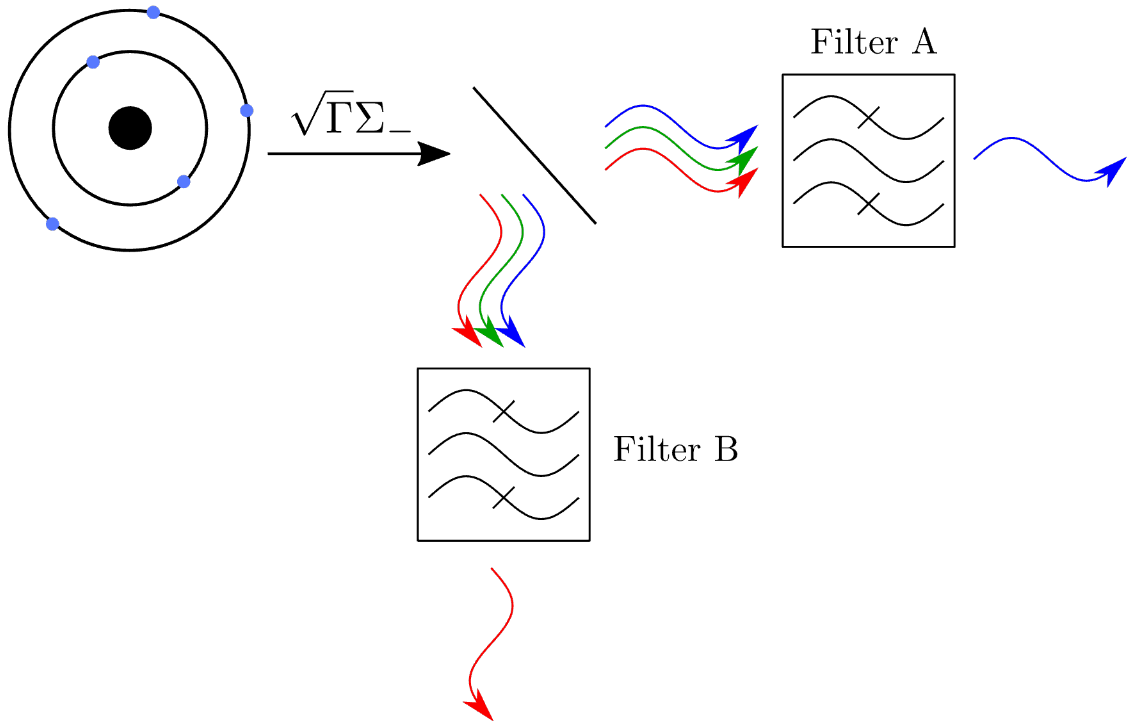

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THE MULTI-MODE ARRAY FILTER

Master equation

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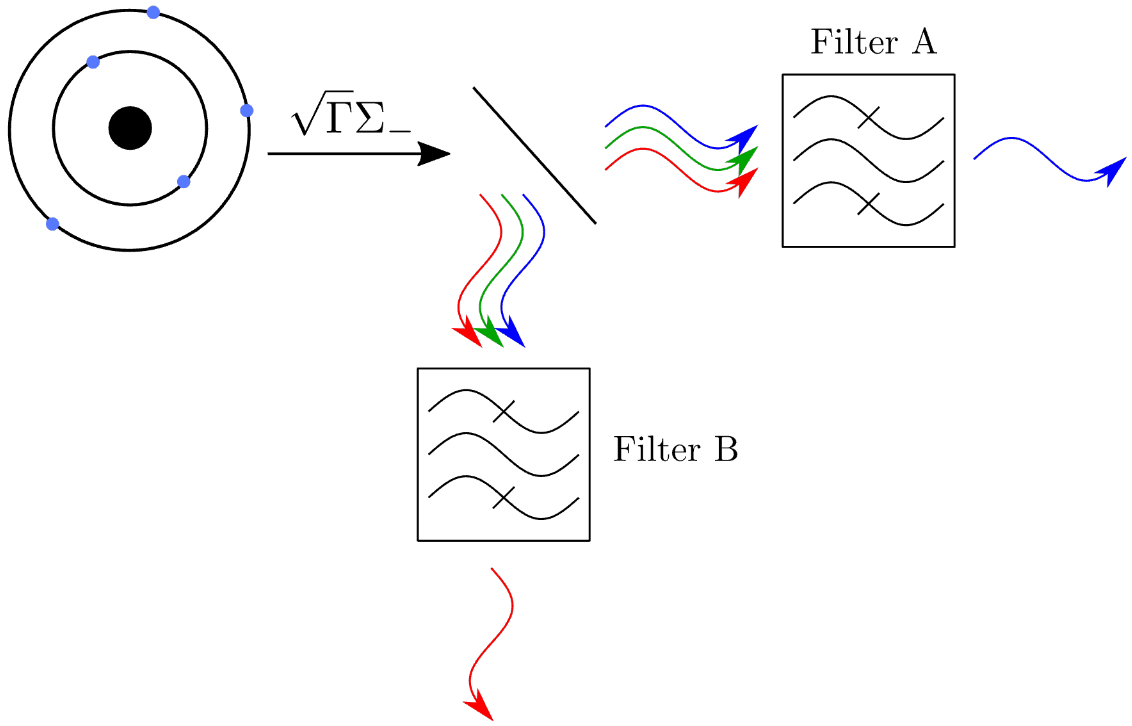
$$-i\sum_{j=-N}^{N}\Delta\omega_j^{(a)}a_j^\dagger a_j + \frac{\kappa}{2}\sum_{j=-N}^{N}\Lambda(a_j)\rho$$ Array Filter A

$$-\sum_{j=-N}^{N}\mathcal{E}_j\left(a_j^\dagger\Sigma_-\rho - \Sigma_-\rho a_j^\dagger\right) - \sum_{j=-N}^{N}\mathcal{E}_j^*\left(\rho\Sigma_+a_j - a_j\rho\Sigma_+\right)$$

$$-i\sum_{j=-N}^{N}\Delta\omega_j^{(b)}b_j^\dagger b_j + \frac{\kappa}{2}\sum_{j=-N}^{N}\Lambda(b_j)\rho$$ Array Filter B

$$-\sum_{j=-N}^{N}\mathcal{E}_j\left(b_j^\dagger\Sigma_-\rho - \Sigma_-\rho b_j^\dagger\right) - \sum_{j=-N}^{N}\mathcal{E}_j^*\left(\rho\Sigma_+b_j - b_j\rho\Sigma_+\right)$$



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THE MULTI-MODE ARRAY FILTER

Master equation

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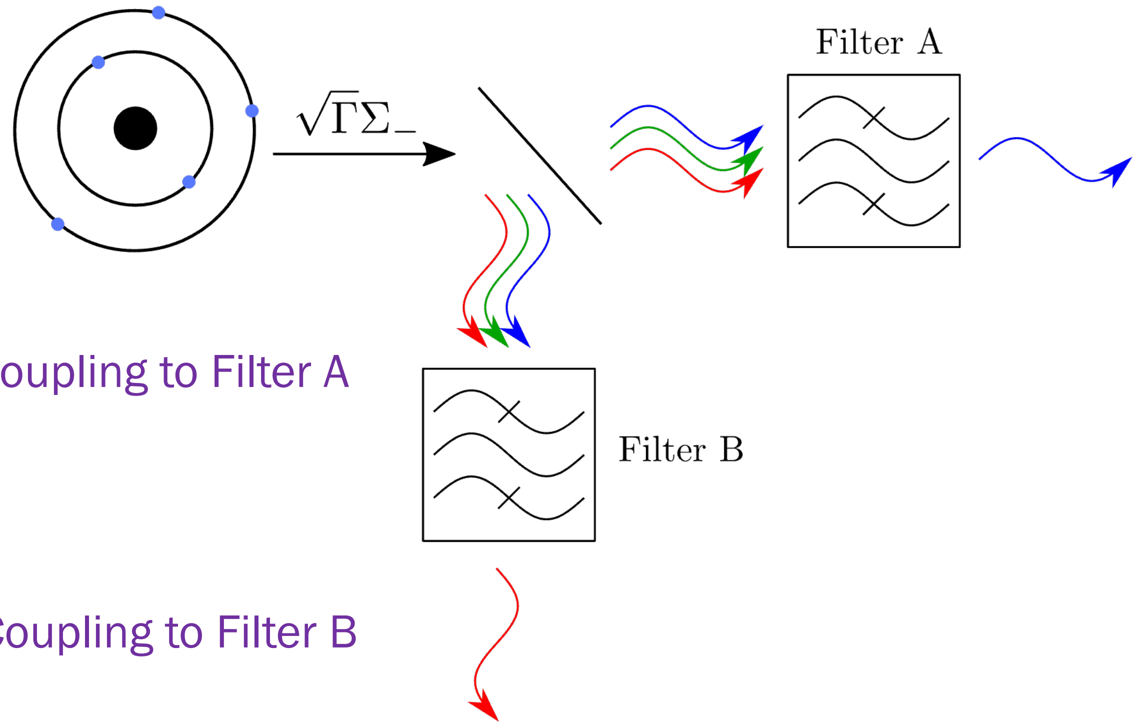
$$- \sum_{j=-N}^{N} \mathcal{E}_j \left(a_j^\dagger \Sigma_- \rho - \Sigma_- \rho a_j^\dagger \right) - \sum_{j=-N}^{N} \mathcal{E}_j^* \left(\rho \Sigma_+ a_j - a_j \rho \Sigma_+ \right)$$ Cascaded coupling to Filter A

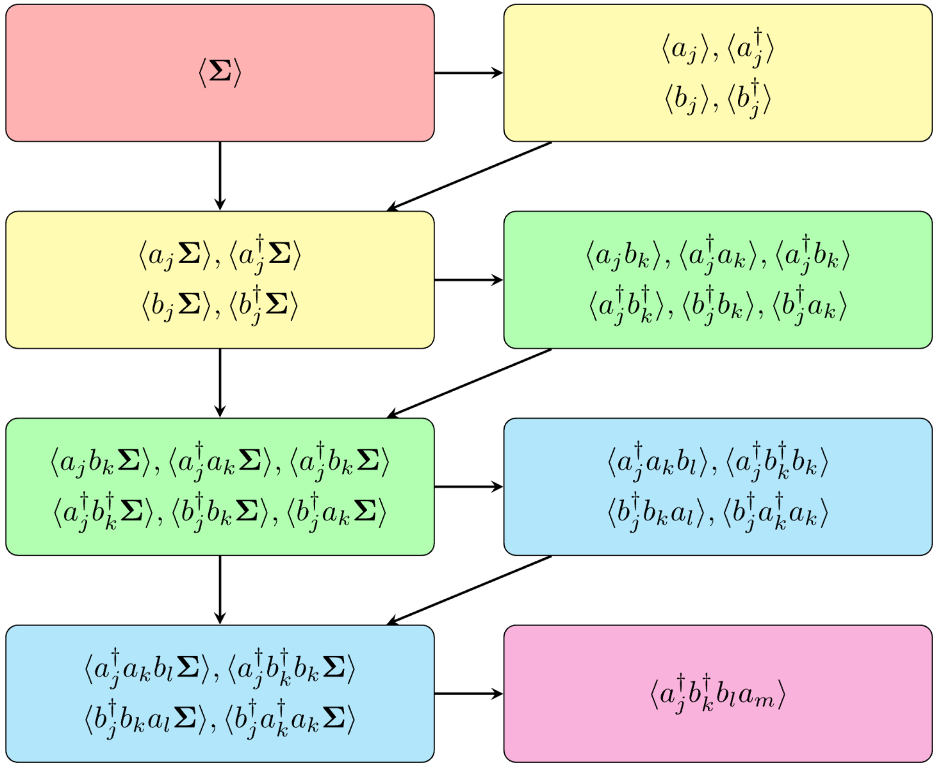
$$- i \sum_{j=-N}^{N} \Delta\omega_j^{(b)} b_j^\dagger b_j + \frac{\kappa}{2} \sum_{j=-N}^{N} \Lambda(b_j)\rho$$ Array Filter B

$$- \sum_{j=-N}^{N} \mathcal{E}_j \left(b_j^\dagger \Sigma_- \rho - \Sigma_- \rho b_j^\dagger \right) - \sum_{j=-N}^{N} \mathcal{E}_j^* \left(\rho \Sigma_+ b_j - b_j \rho \Sigma_+ \right)$$ Cascaded Coupling to Filter B



$$\Lambda(X)\bullet = 2X \bullet X^\dagger - X^\dagger X \bullet - \bullet X^\dagger X$$

THE MULTI-MODE ARRAY FILTER



- Frequency-filtered second-order correlation function:

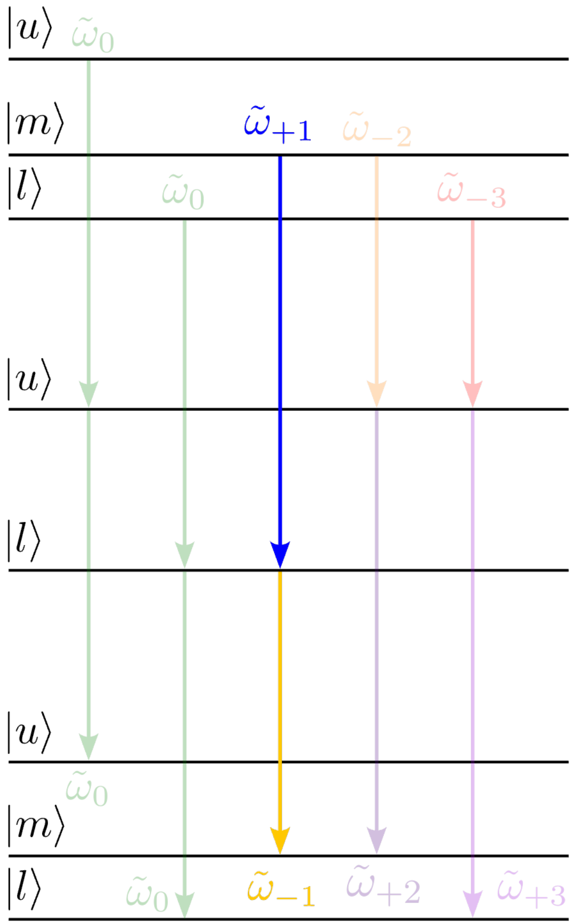
$$g^{(2)}(\alpha, 0; \beta, \tau) = \frac{\langle A^\dagger(0) B^\dagger B(\tau) A(0) \rangle_{ss}}{\langle A^\dagger A \rangle_{ss} \langle B^\dagger B \rangle_{ss}}$$

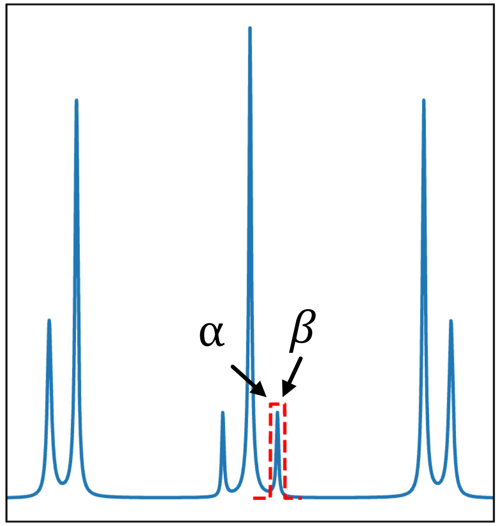
- Collective mode annihilation operators

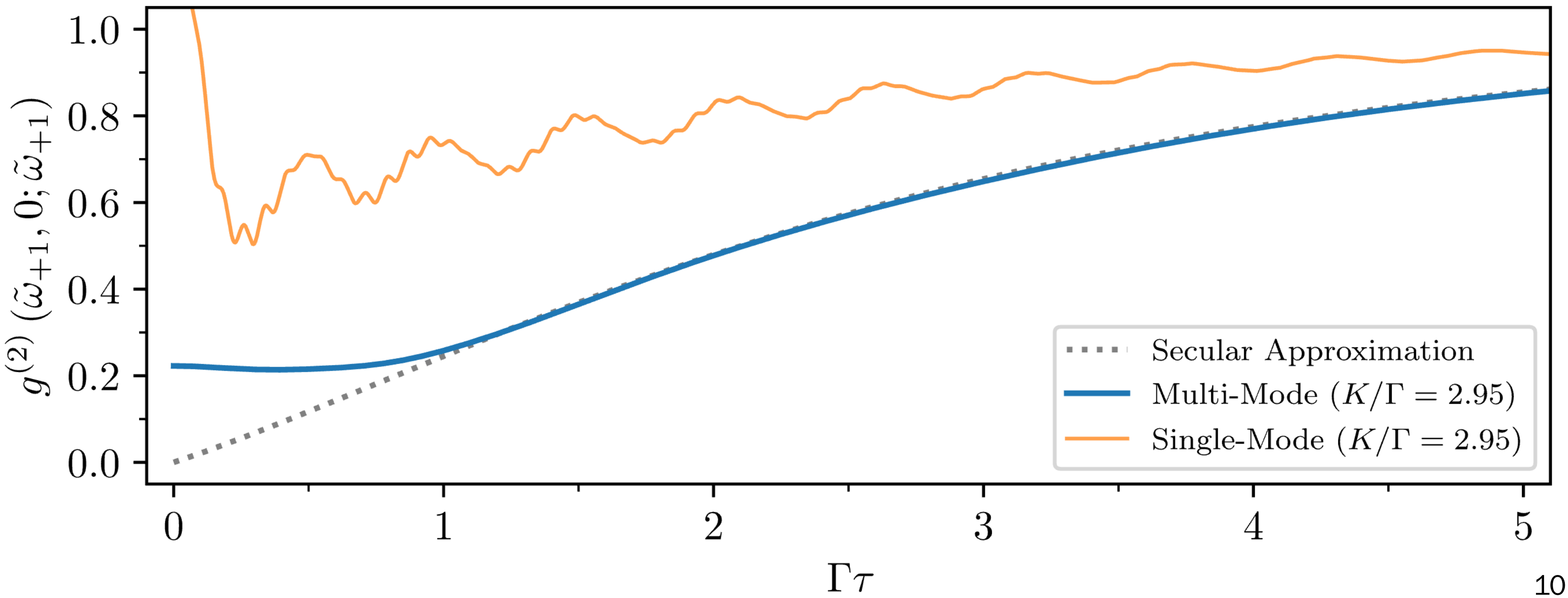
$$A = \sum_{j=-N}^{N} a_j, \quad B = \sum_{j=-N}^{N} b_j$$

- Moment equations – efficient method for calculating

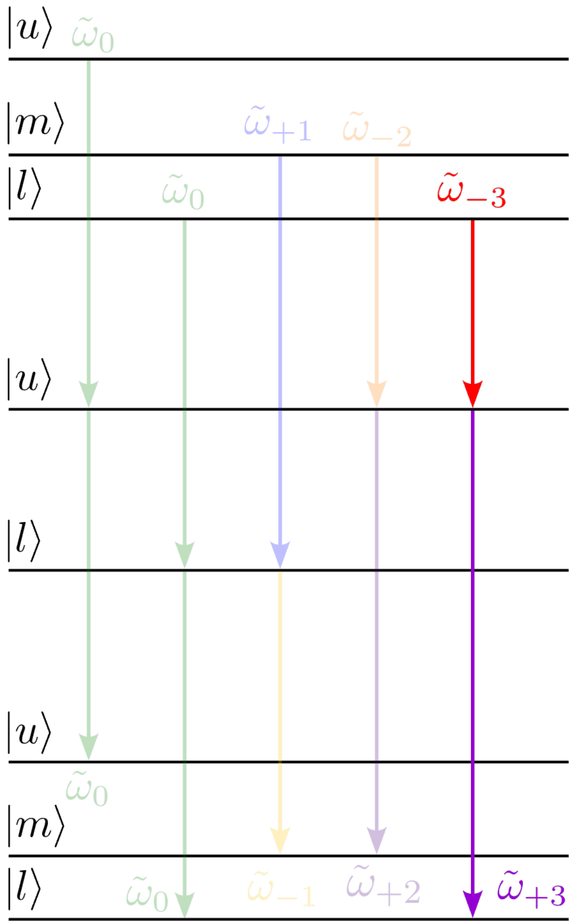
FREQUENCY-FILTERED AUTO-CORRELATIONS

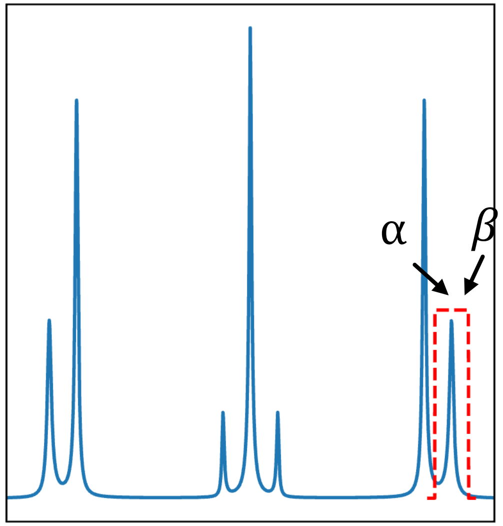


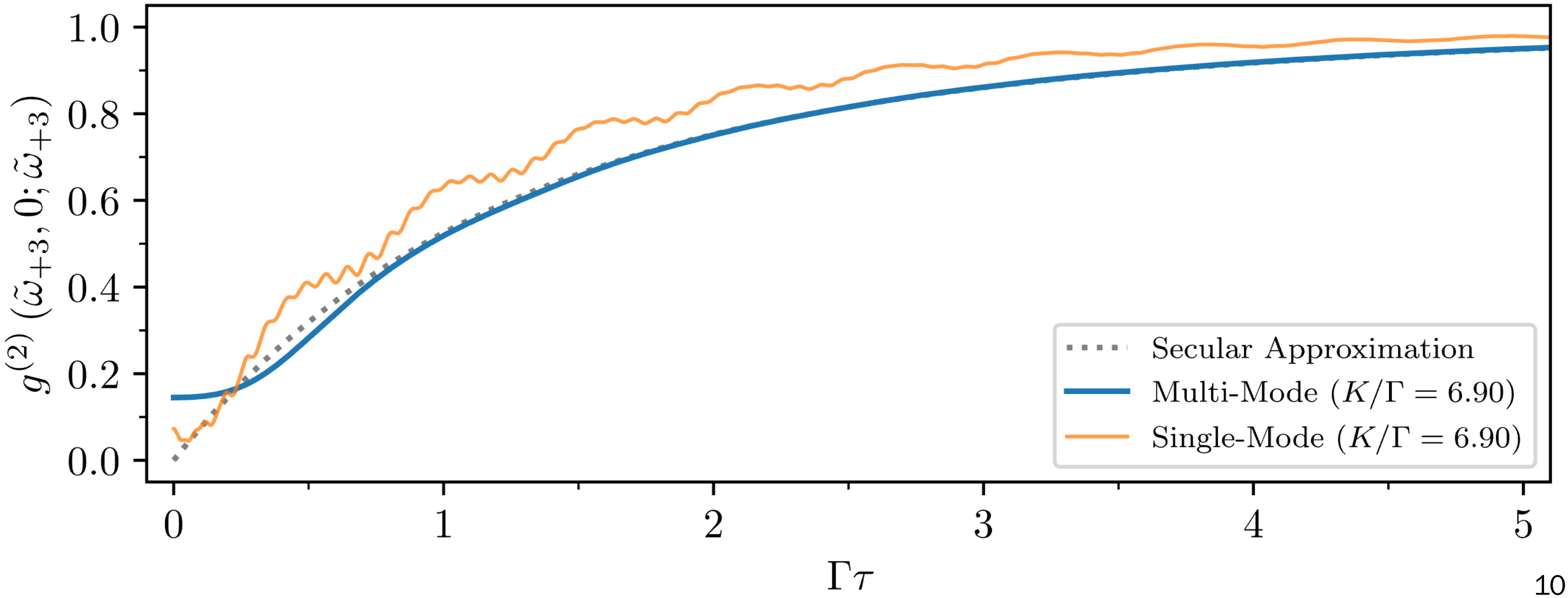




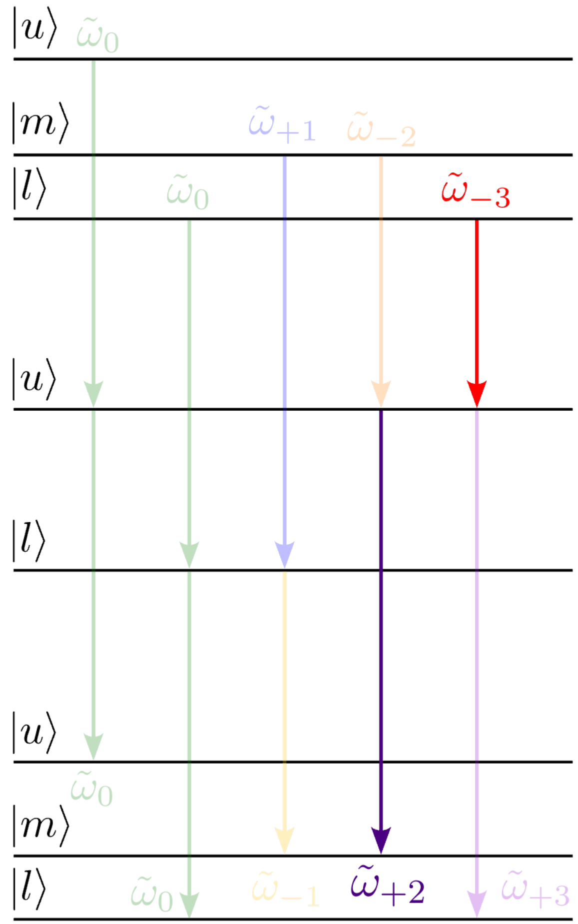
FREQUENCY-FILTERED AUTO-CORRELATIONS

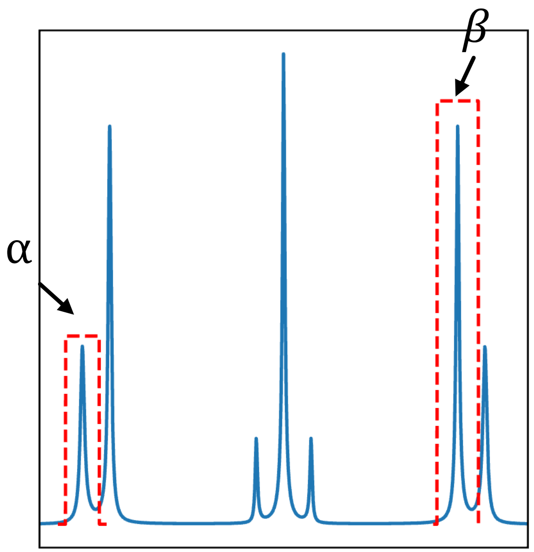




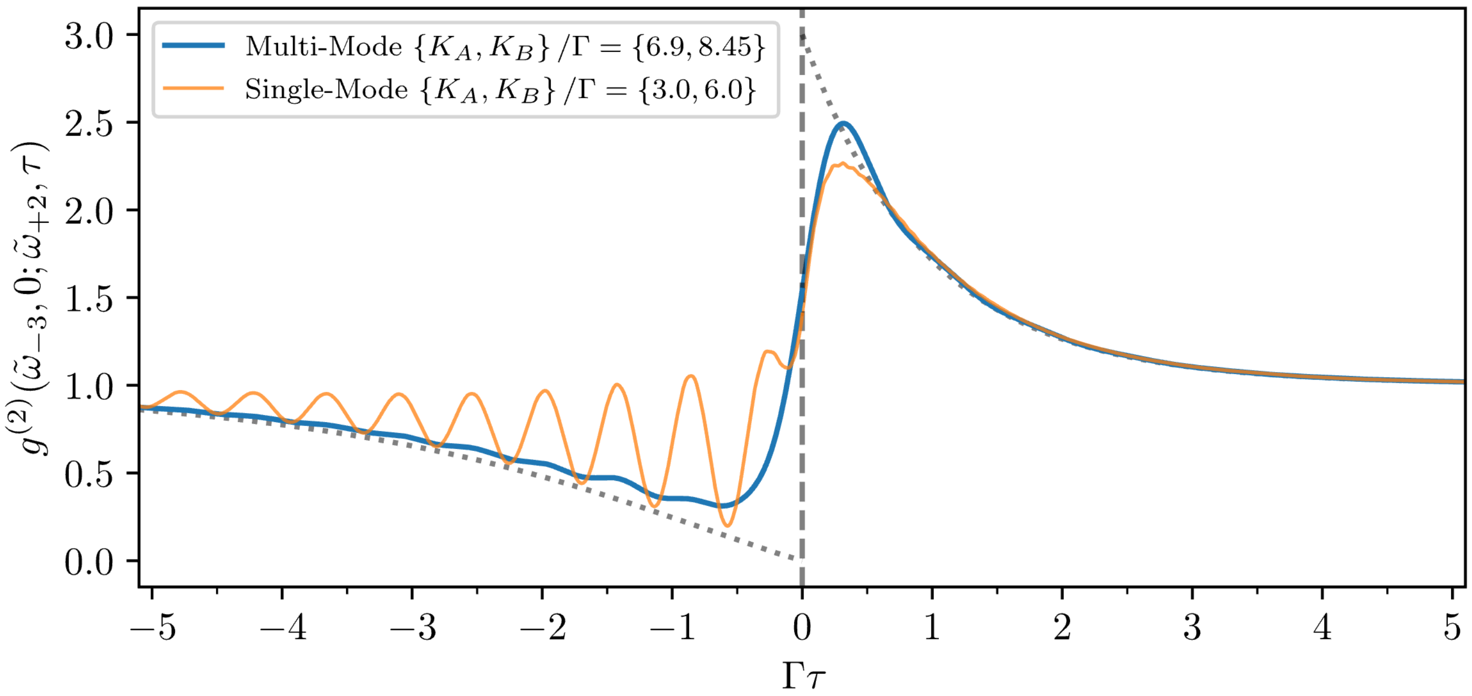


FREQUENCY-FILTERED CROSS-CORRELATIONS

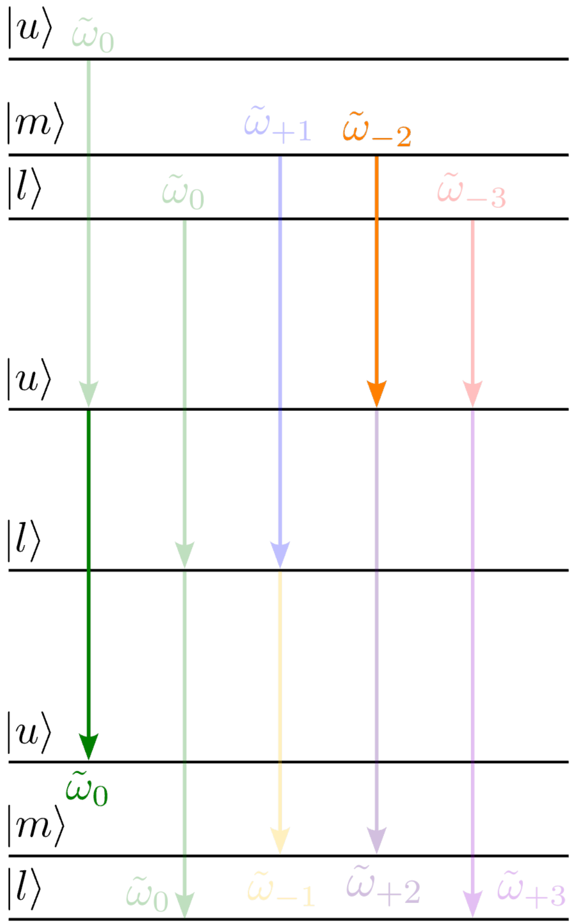




Interference of time-ordering
PRL **67**, 2443 (1991) PRA **45**, 8045 (1992)



FREQUENCY-FILTERED CROSS-CORRELATIONS



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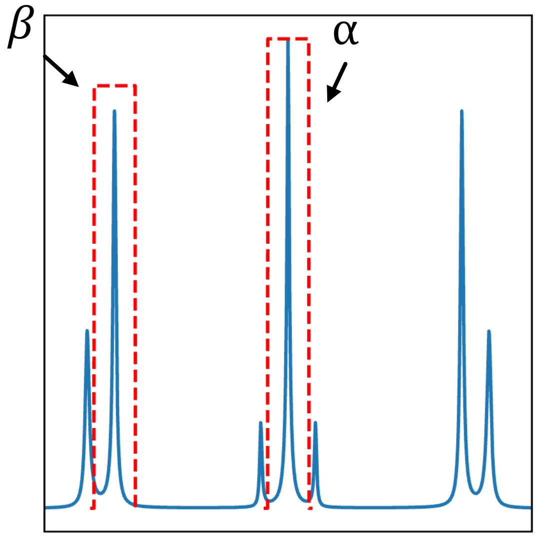
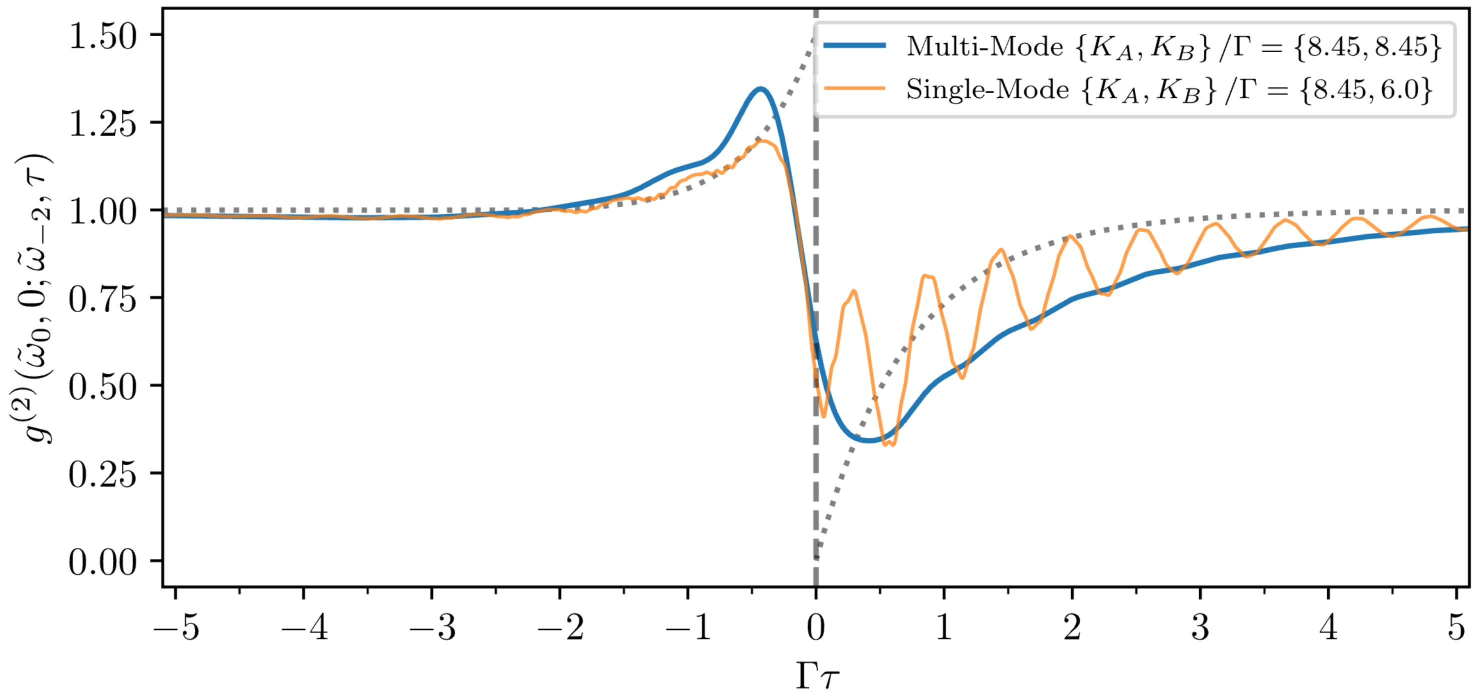
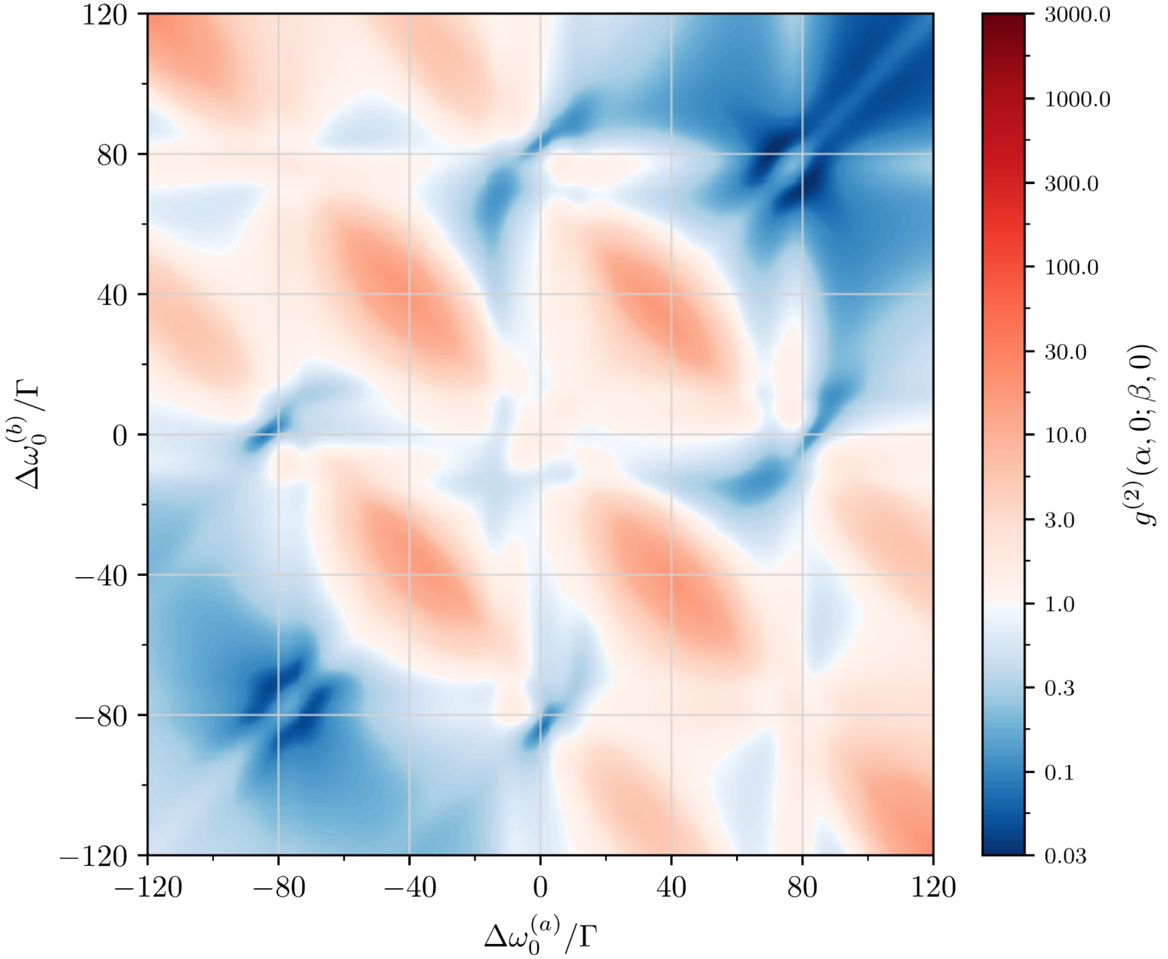




LANDSCAPES OF PHOTON CORRELATIONS

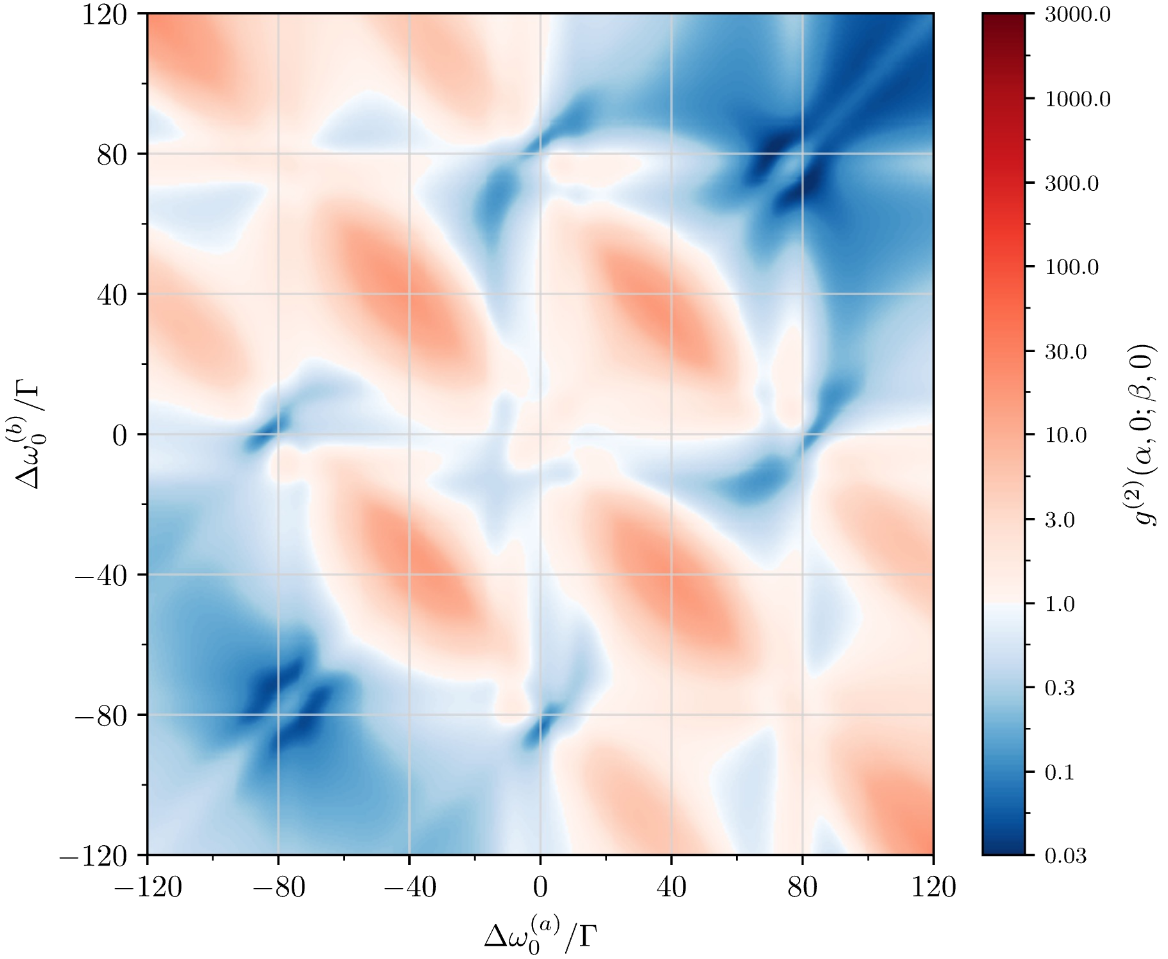


Single-Mode filter with $\{K_A, K_B\}/\Gamma = \{4, 4\}$

Initial correlation values

- Red – correlated / bunching
- White – uncorrelated / random
- Blue – anti-correlated / antibunched

LANDSCAPES OF PHOTON CORRELATIONS



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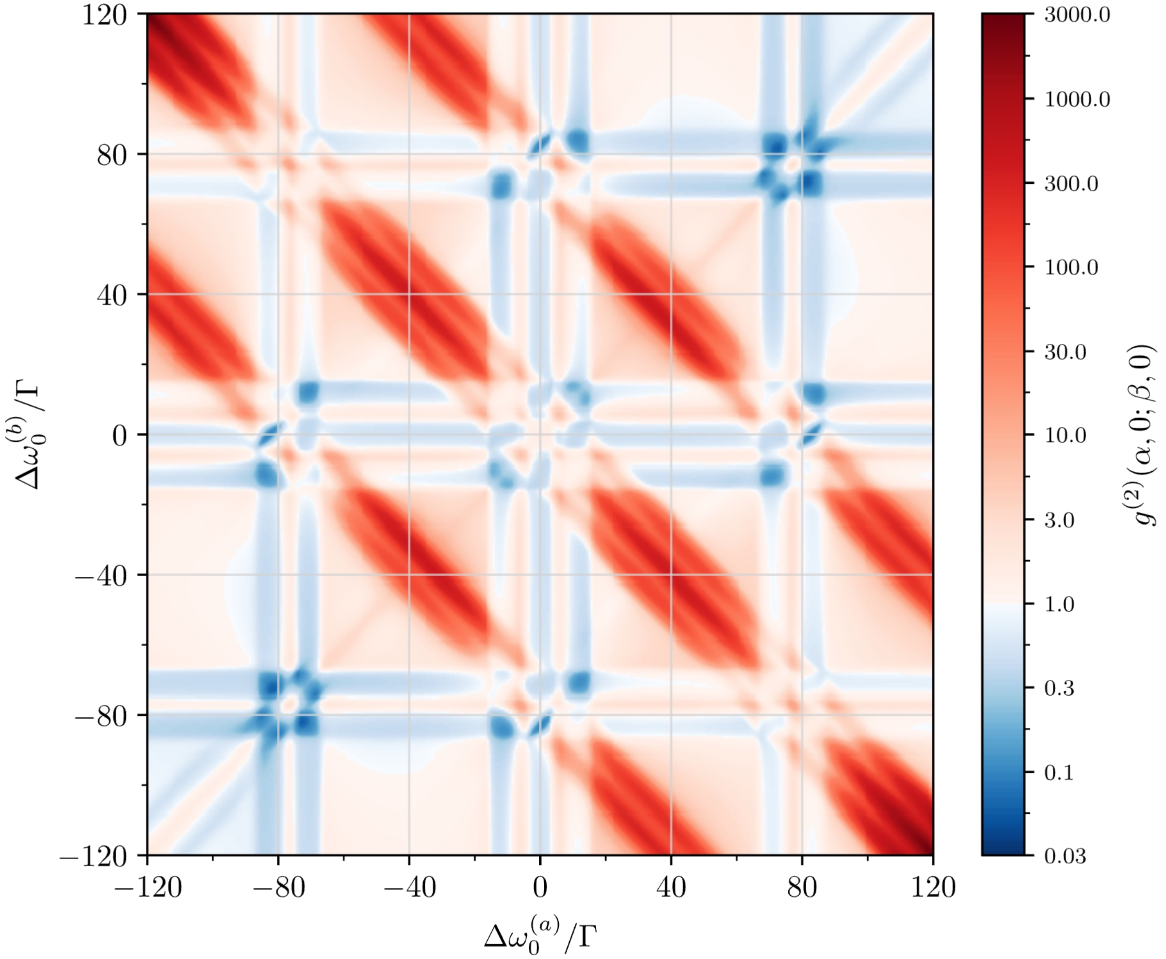


Multi-mode array filter with $\{K_A, K_B\}/\Gamma = \{4, 4\}$

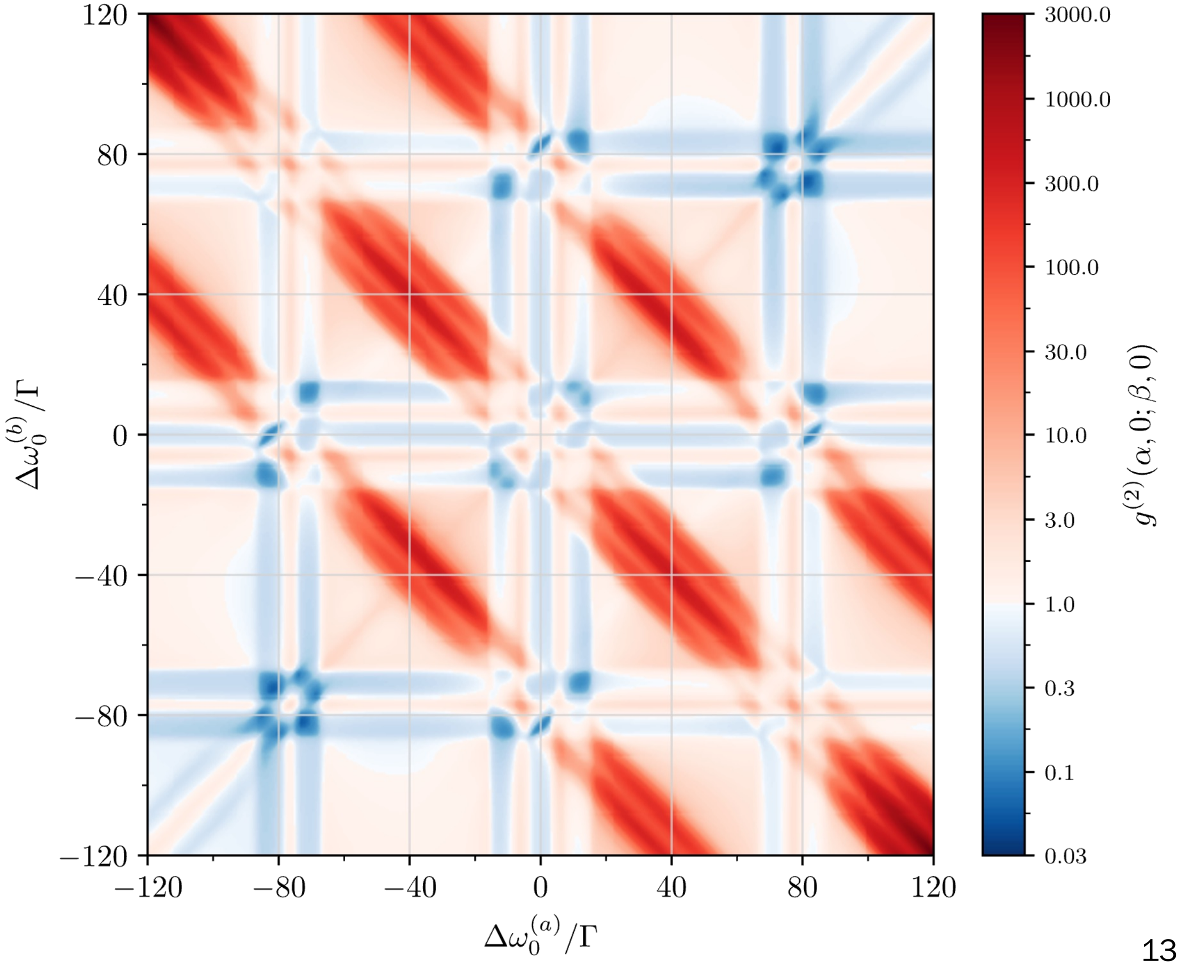
CONCLUSIONS

Three-level ladder type atom

- Unique fluorescence spectrum
- New and interesting regions of photon correlations

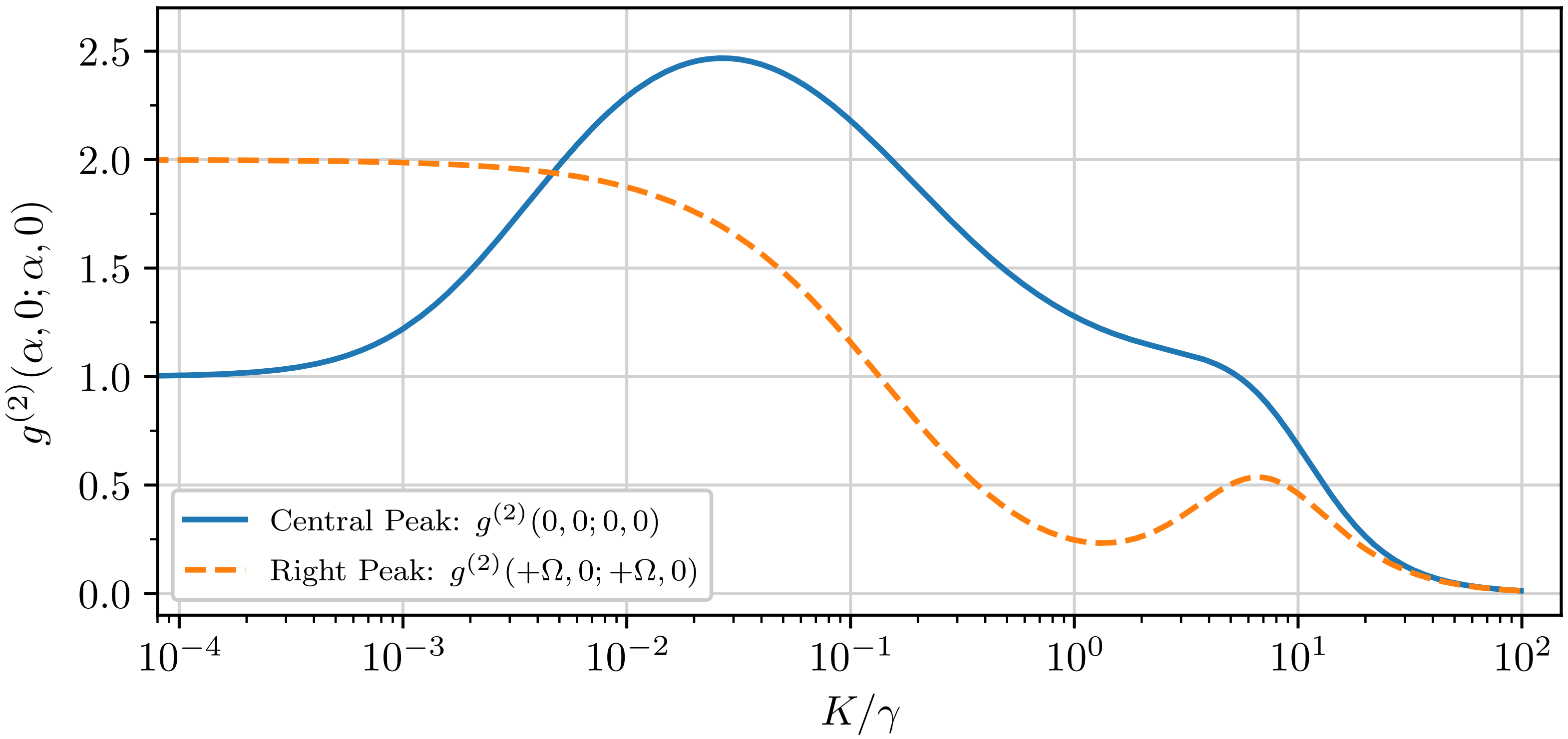
Multi-mode array filter

- Sharper frequency response can find for new regions of photon correlations
- A large system with an extremely efficient method of calculations
- *Probably* an experimental nightmare



NARROW BANDWIDTHS – SINGLE-MODE

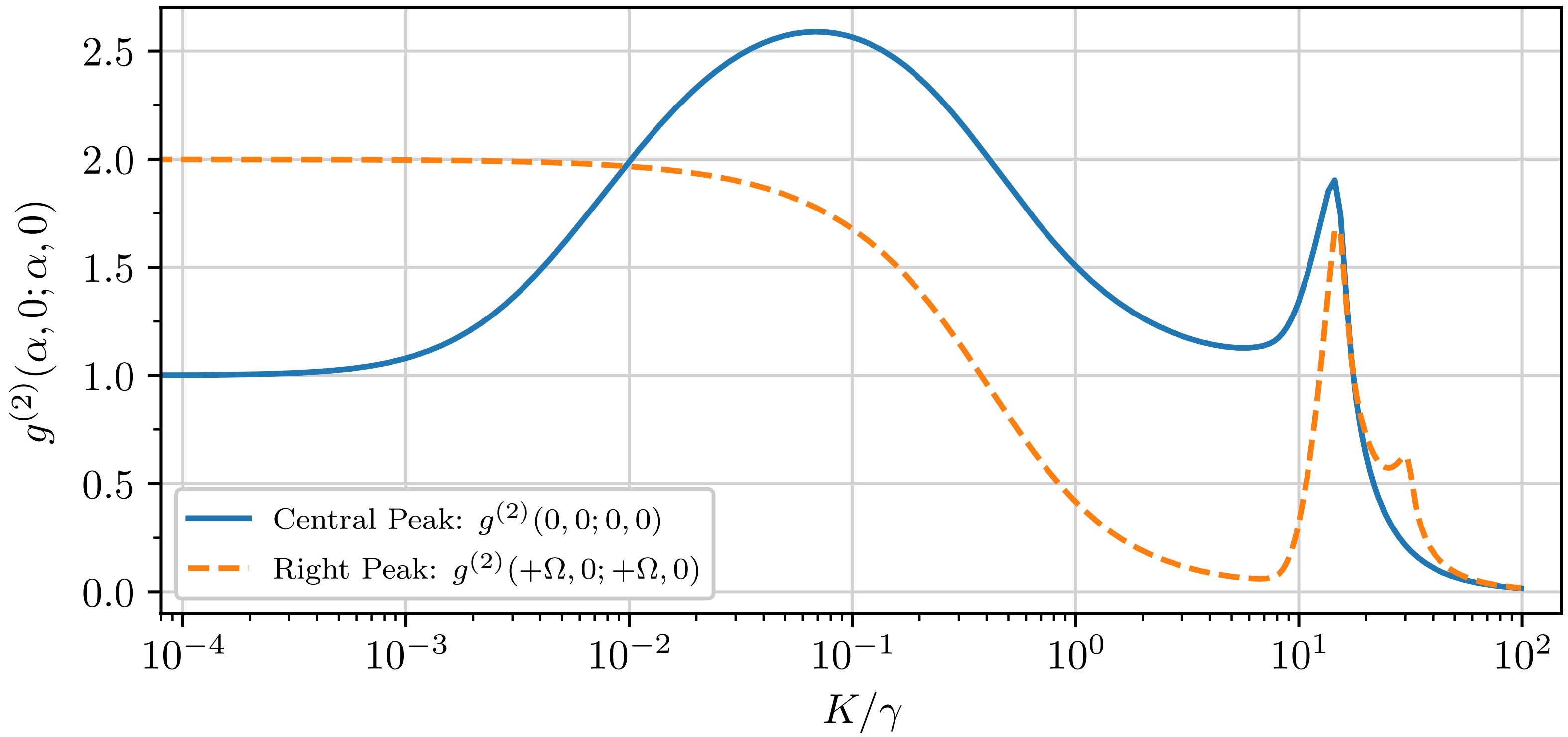
- What happens when we *decrease* the bandwidth?
- Large bandwidth = antibunching
- Vanishing bandwidth = uncorrelated (central peak), thermal (right peak)



Single-mode auto-correlations

NARROW BANDWIDTHS – MULTI-MODE

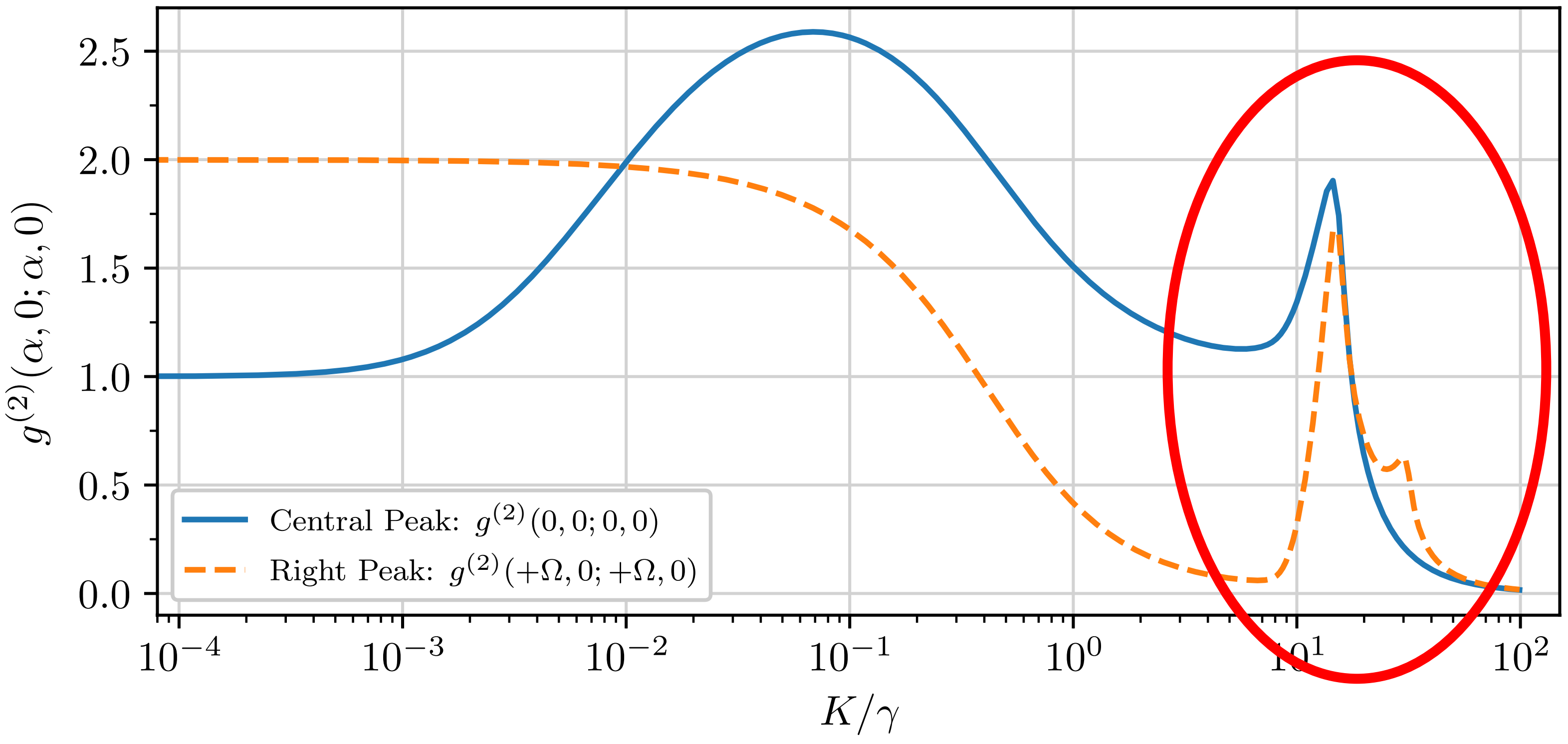
- Similar results to single-mode for extreme bandwidths
- Large bunching appearing when bandwidth is close to peak separation (K ≈ Ω)
- Only visible with improved frequency isolation of multi-mode array filter



Multi-mode auto-correlations

NARROW BANDWIDTHS – MULTI-MODE

- Similar results to single-mode for extreme bandwidths
- Large bunching appearing when bandwidth is close to peak separation ($K \approx \Omega$)
- Only visible with improved frequency isolation of multi-mode array filter



Multi-mode auto-correlations