A Better Method for Filtered Photon Correlations

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1 July, 2021

Introduction

- Single-mode Lorentzian filters have trade off between frequency cut-off and temporal response.
- Want a better approximation to a box filter that is effective and efficient.

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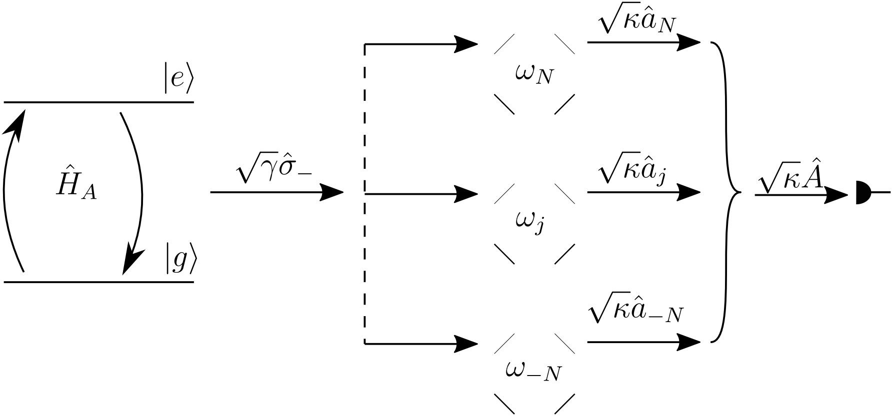
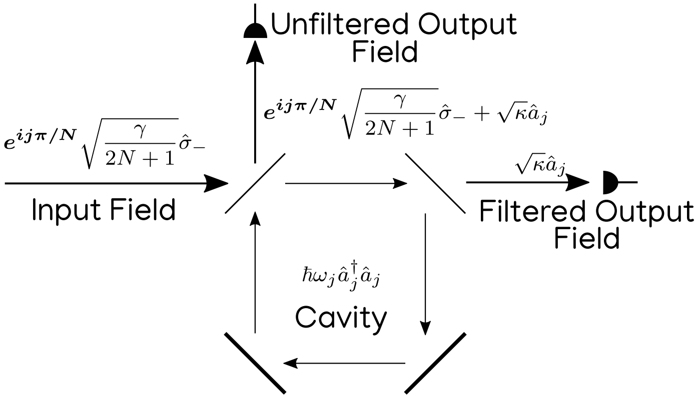
Schematic



$|e\rangle$
$|g\rangle$
$\hat{H}_A$
$\sqrt{\gamma}\hat{\sigma}_-$
ω_N
ω_j
ω_{-N}
$\sqrt{\kappa}\hat{a}_N$
$\sqrt{\kappa}\hat{a}_j$
$\sqrt{\kappa}\hat{a}_{-N}$
$\sqrt{\kappa}\hat{A}$

Schematic



Unfiltered Output Field
$e^{ij\pi/N}\sqrt{\frac{\gamma}{2N+1}}\hat{\sigma}_- + \sqrt{\kappa}\hat{a}_j$
$e^{ij\pi/N}\sqrt{\frac{\gamma}{2N+1}}\hat{\sigma}_-$
Input Field
$\sqrt{\kappa}\hat{a}_j$
Filtered Output Field
$\hbar\omega_j\hat{a}_j^\dagger\hat{a}_j$
Cavity

Photon Correlations

- Collective mode operator

$$\hat{A} = \sum_{j=-N}^{N} \hat{a}_j .$$

- Filtered second-order correlation

$$g_{ss}^{(2)}(\tau) = \frac{\langle \hat{A}^\dagger(0)\hat{A}^\dagger \hat{A}(\tau)\hat{A}(0)\rangle_{ss}}{\langle \hat{A}^\dagger \hat{A}\rangle_{ss}^2} .$$

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Solving the Master Equation

- Density operator $\hat{\rho} = |\psi\rangle\langle\psi|$ and state ($i = g, e,$ and $n = 0, 1, 2, \dots$)

$$|\psi\rangle = |i = g, e\rangle_{\text{Atom}} \otimes |n\rangle_{-N} \otimes \cdots |n\rangle_j \otimes \cdots \otimes |n\rangle_N \, .$$

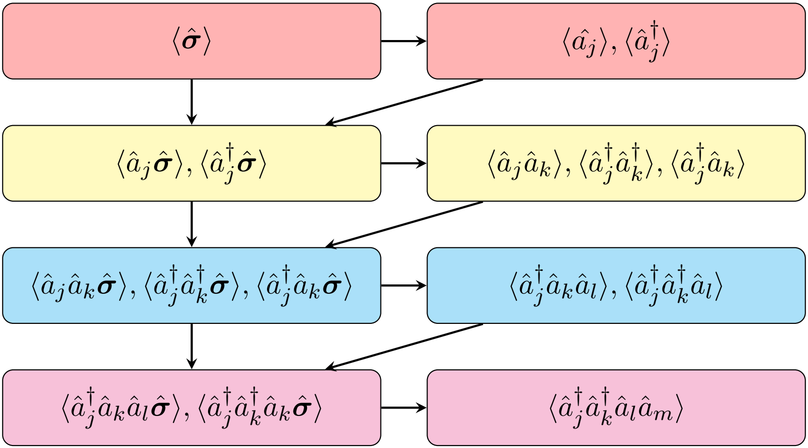
- Makes for a LARGE system.

- For 41 modes ($N = 20$), ≈ 20 hours to solve.

- Instead solve for operator averages

$$\frac{\mathrm{d}}{\mathrm{d}t}\langle\hat{X}(t)\rangle = \mathrm{tr}_S\left[\hat{X}\frac{\mathrm{d}\hat{\rho}}{\mathrm{d}t}\right] .$$

Operator Moment Equations



Quantum Regression

If

$$\frac{\mathrm{d}}{\mathrm{d}t}\langle\hat{O}(t)\rangle = \boldsymbol{M}\langle\hat{O}(t)\rangle,$$

then

$$\frac{\mathrm{d}}{\mathrm{d}\tau}\langle\hat{A}^\dagger(0)\hat{O}(\tau)\hat{A}(0)\rangle = \boldsymbol{M}\langle\hat{A}^\dagger(0)\hat{O}(\tau)\hat{A}(0)\rangle.$$

Filtered photon-correlations:

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}\tau}\langle\hat{A}^\dagger(0)\hat{a}_j^\dagger\hat{a}_k(\tau)\hat{A}(0)\rangle = &-\left[2\kappa - i\left(\omega_j - \omega_k\right)\right]\langle\hat{A}^\dagger(0)\hat{a}_j^\dagger\hat{a}_k(\tau)\hat{A}(0)\rangle \\ &- \mathcal{E}_j^*\langle\hat{A}^\dagger(0)\hat{\sigma}_+\hat{a}_k(\tau)\hat{A}(0)\rangle \\ &- \mathcal{E}_k\langle\hat{A}^\dagger(0)\hat{\sigma}_-\hat{a}_j^\dagger(\tau)\hat{A}(0)\rangle,\end{aligned}$$

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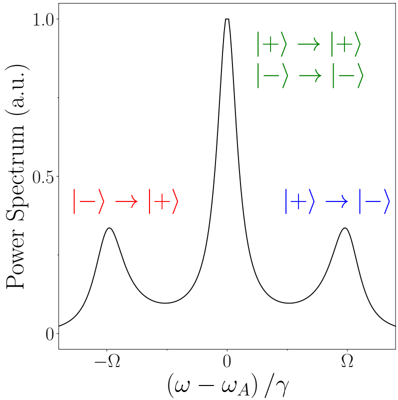
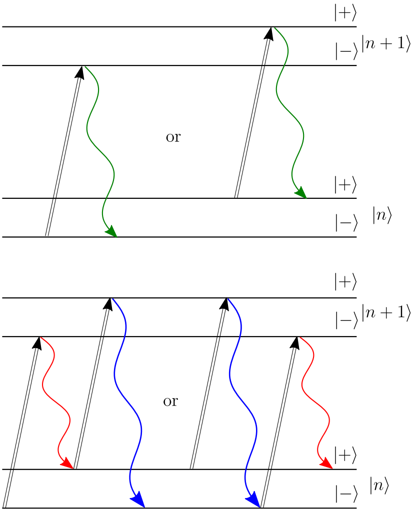
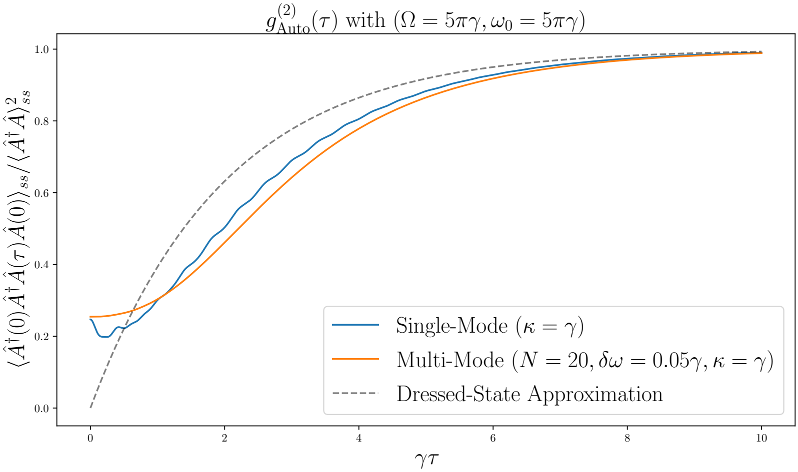
Two-Level Resonance Fluorescence

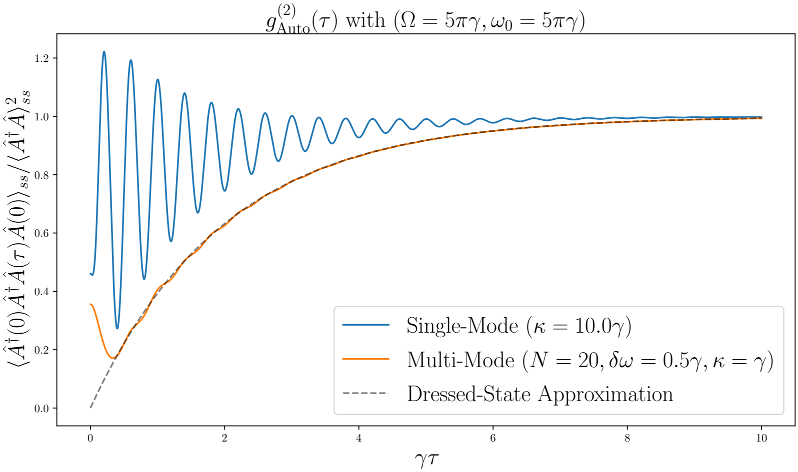




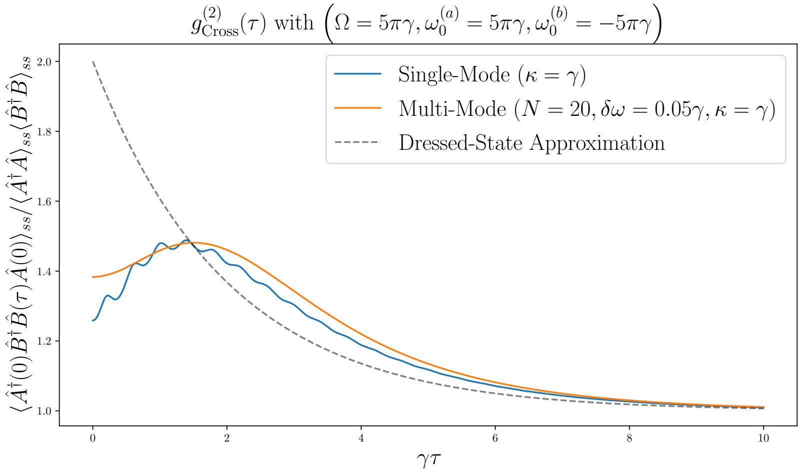
Auto-Correlation: Bandwidth = γ



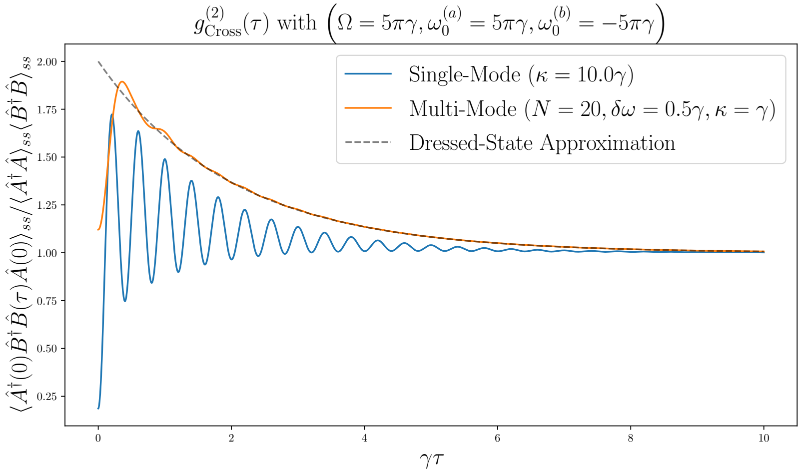
Auto-Correlation: Bandwidth = 10γ



Cross-Correlation: Bandwidth $= \gamma$



Cross-Correlation: Bandwidth $= 10\gamma$



Conclusion

- We have a quick and effective method.
- Solving master equation $\approx$ 20 hours $\Longrightarrow$ moment equations $\approx$ 1 second.
- Easier to add more filters for higher-order correlations.
- Structure of moment equations makes it easy to change source system.

Conclusion

Ngā mihi nui ki a koutou!

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