MULTI-MODE ARRAY FILTERING OF RESONANCE FLUORESCENCE

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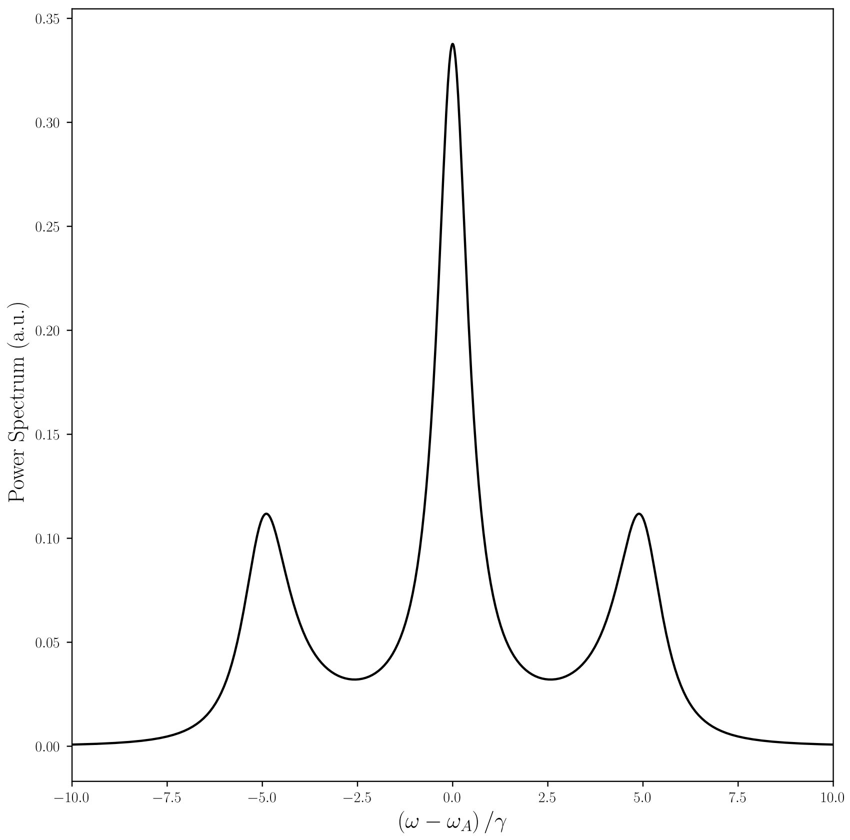
NEW ZEALAND

INTRODUCTION

- Mollow’s paper on resonance fluorescence (1969)
- Antibunching of two-level atoms
- Interest in correlating photons from side-peaks
- Can we do it better?



OVERVIEW

1. **Resonance Fluorescence** – what will be filtered
2. **Filtering (of)** – a simple filtering case
3. **Multi-Mode Array** – our proposed model and results

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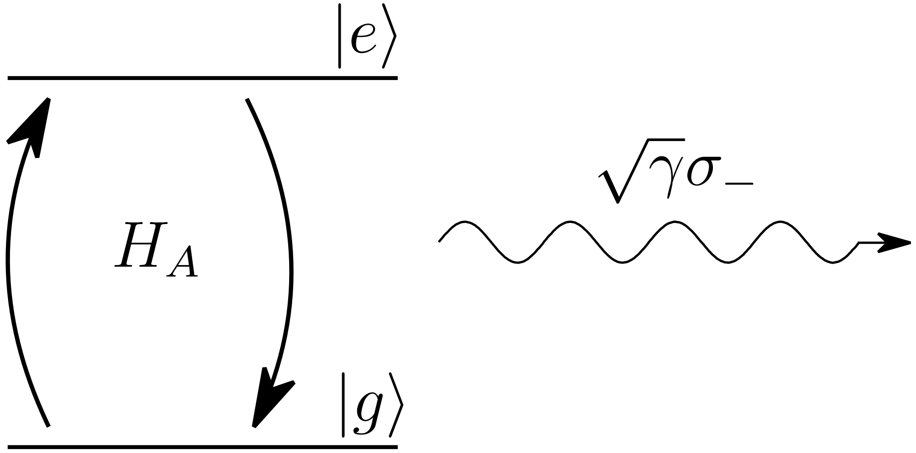
DRIVEN TWO-LEVEL ATOM

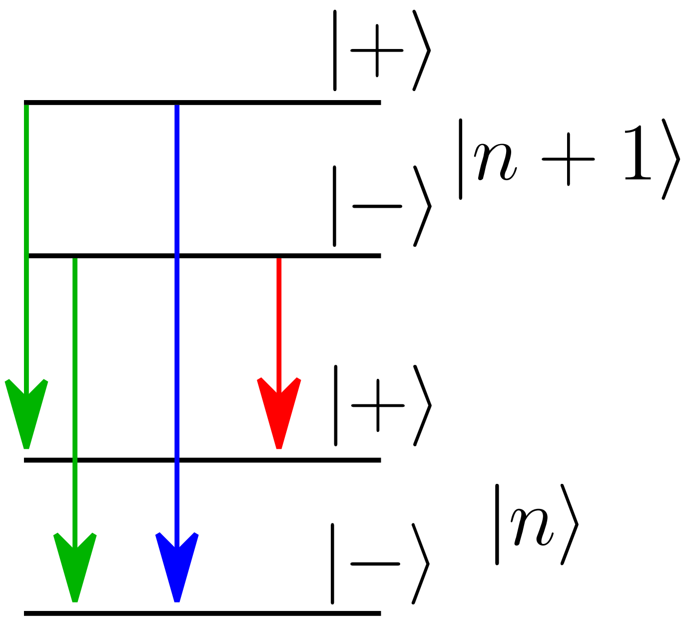


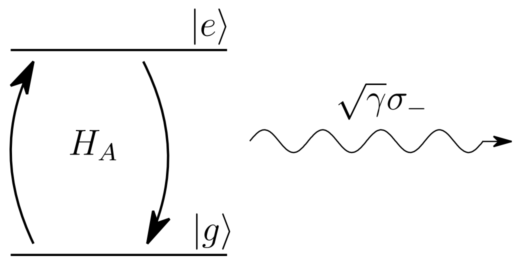
Hamiltonian: $H_A = \hbar\frac{\Omega}{2}(\sigma_+ + \sigma_-)$

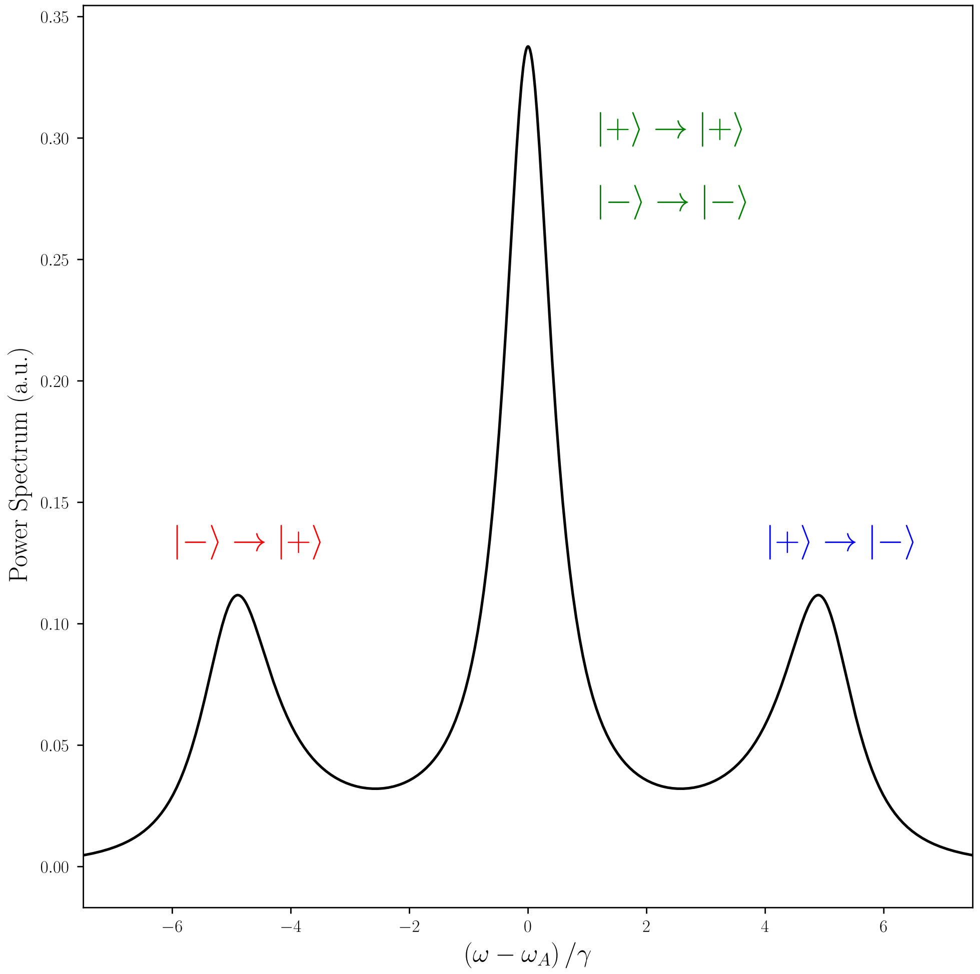
Master Equation: $\frac{d\rho}{dt} = \frac{1}{i\hbar}[H_A, \rho] + \frac{\gamma}{2}(2\sigma_-\rho\sigma_+ - \sigma_+\sigma_-\rho - \rho\sigma_+\sigma_-)$

ρ – Density operator
γ – Atomic decay rate
Ω – Driving amplitude (Rabi frequency)
$\sigma_+ = |e\rangle\langle g|, \sigma_- = |g\rangle\langle e|$ - atomic raising and lowering operators

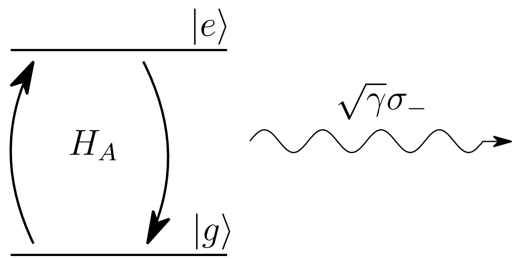
DRESSED STATES



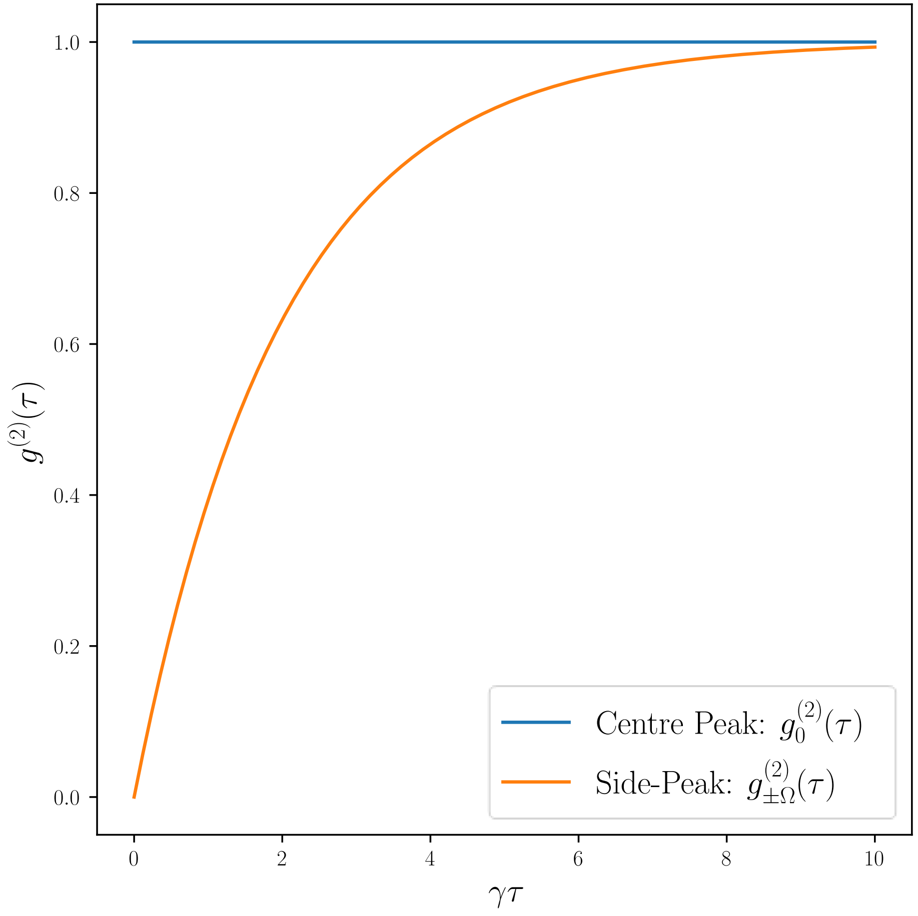




PHOTON CORRELATIONS



- Transform to dressed-state picture
- Centre peak: $g_0^{(2)} = 1.0$
- Side-peaks: $g_{\pm\Omega}^{(2)} = 1.0 - e^{-\frac{\gamma}{2}\tau}$



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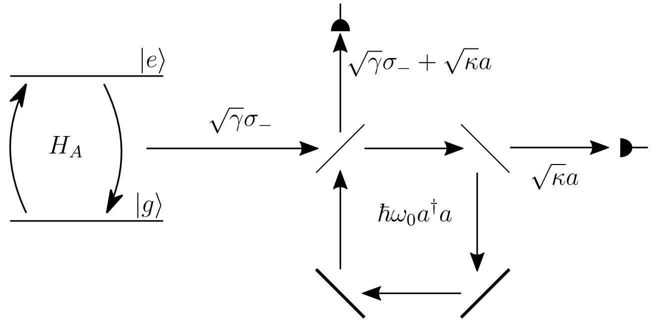
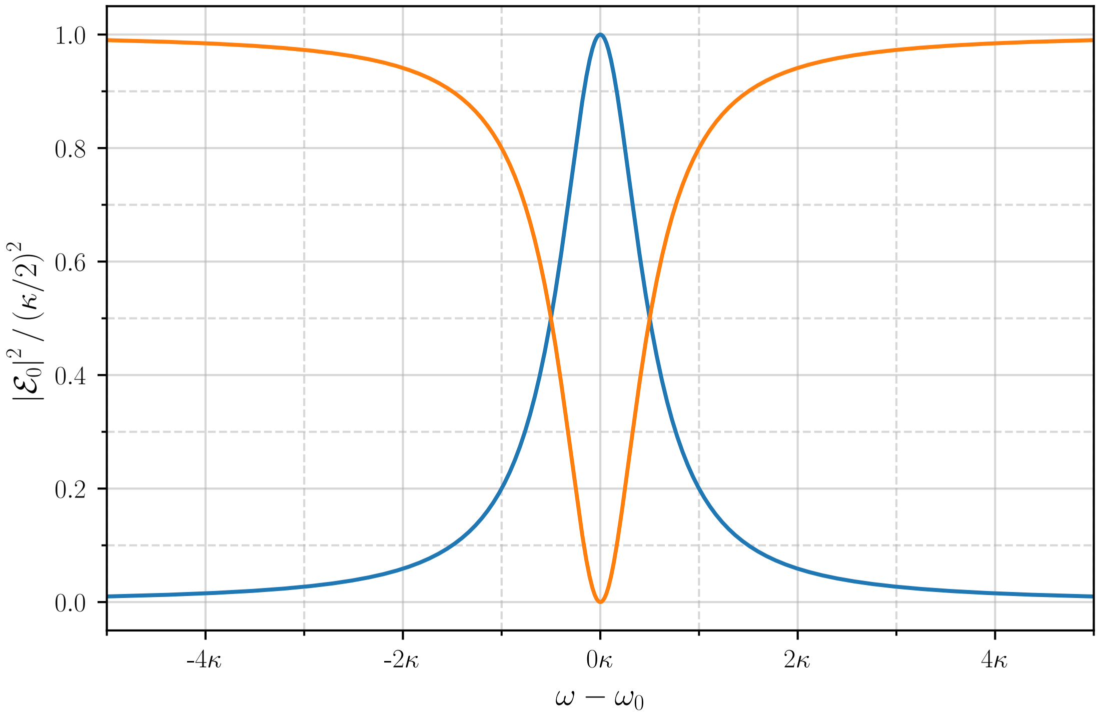


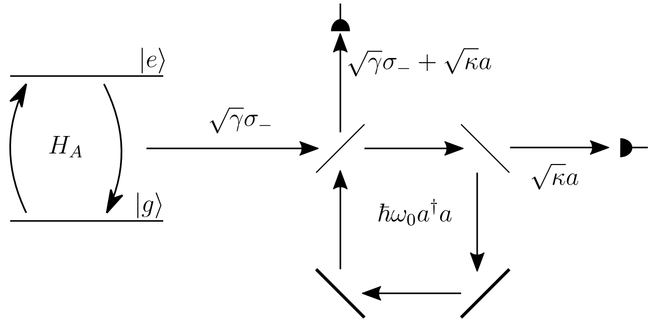
LORENTZIAN FILTER

$$S(\omega) \approx \frac{\mathcal{E}_0}{(\kappa)^2 + (\omega - \omega_0)^2}$$

- Large bandwidth = good temporal response
  BUT poor isolation
- Narrow bandwidth = good peak isolation
  BUT poor temporal response

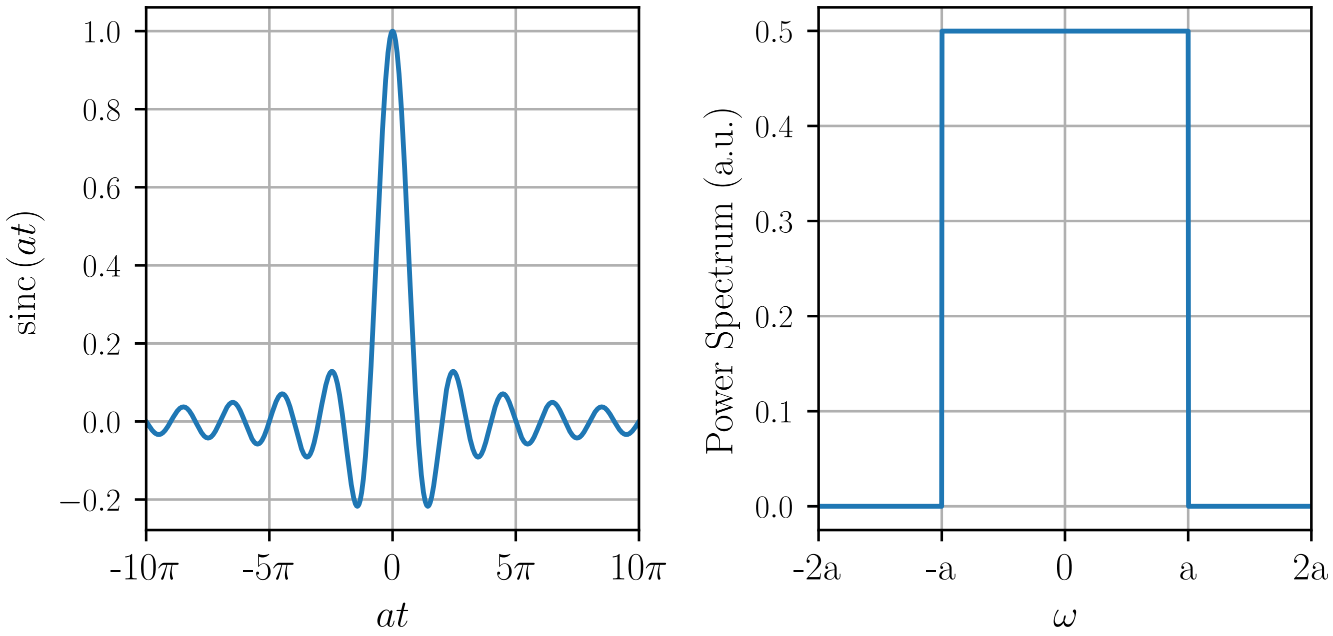


SINC FILTERS



$$\mathrm{sinc}(t) = \frac{\sin(t)}{t}$$

- Fourier transform of a sinc is a rectangle function
- Immediate drop off
- Non-causal, physically unrealisable
- Phase shift to recreate box



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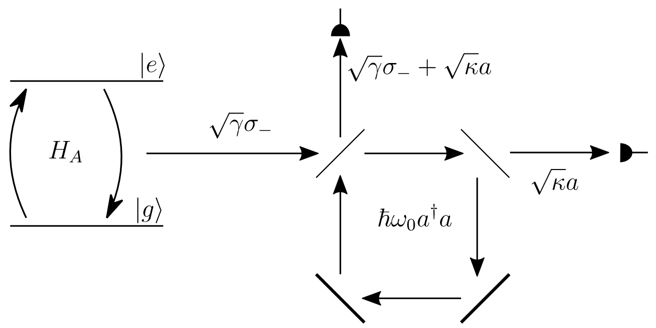

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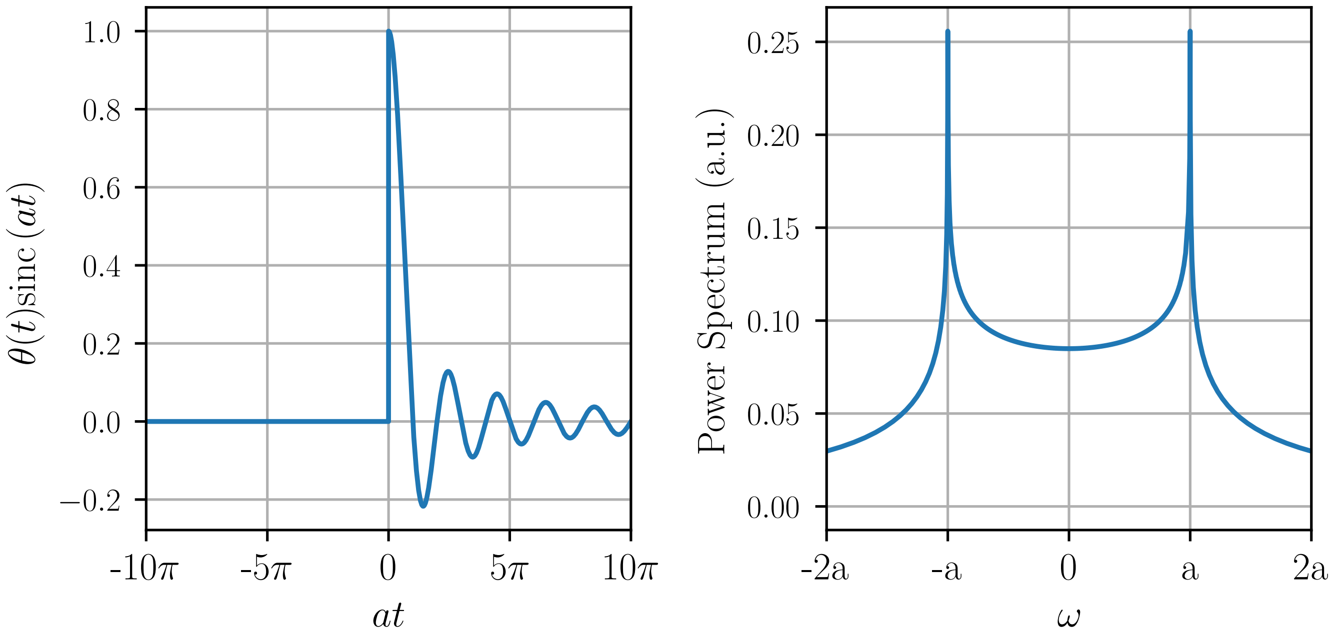

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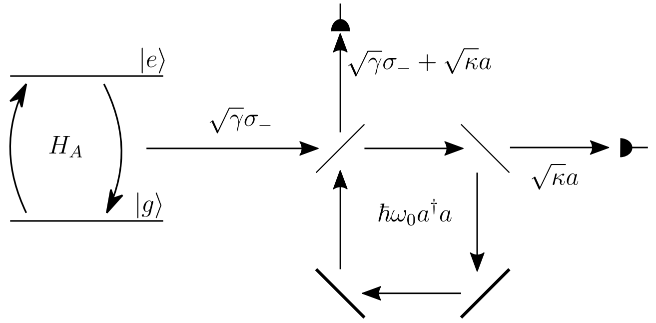

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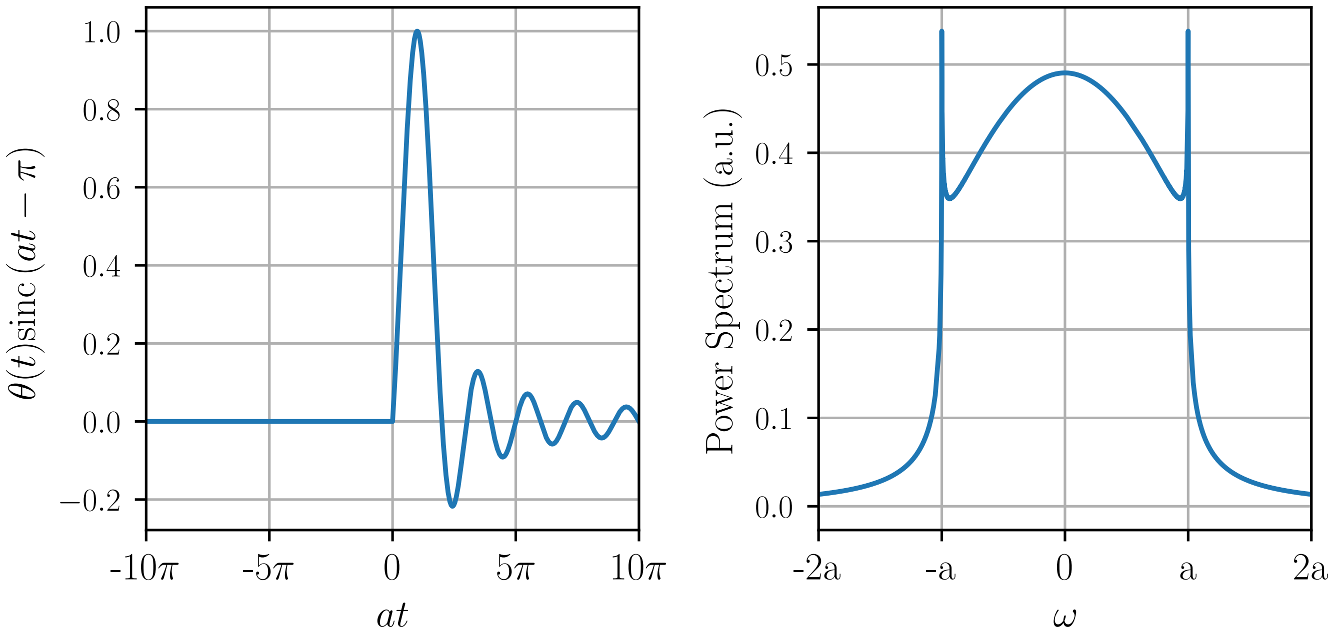

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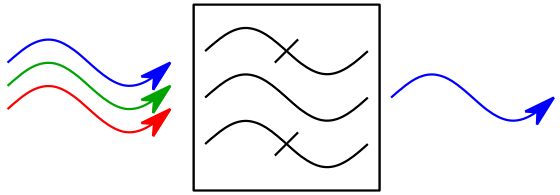


OUR MODEL

Hamiltonian:

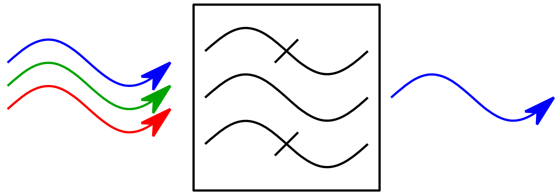
$$H = \hbar\frac{\Omega}{2}(\sigma_+ + \sigma_-) + \hbar\sum_{j=-N}^{N}\omega_j a_j^\dagger a_j + \frac{i\hbar}{2}\sqrt{\frac{\gamma\kappa}{2N+1}}\sum_{j=-N}^{N}\left(e^{-imj\pi/N}a_j\sigma_+ - e^{imj\pi/N}a_j^\dagger\sigma_-\right)$$

Driven atom | Filter Modes | Cascaded system coupling

$\omega_j = \omega_0 + \delta\omega$ - Resonance frequency of j^{th} mode

ω_0 – Central mode resonance frequency
$\delta\omega$ – Mode frequency spacing
m – Integer sets size for phase delay

κ – Cavity decay rate for each mode
$a_j^\dagger, a_j$ – photon creation and annihilation operators for the j^{th} mode



OUR MODEL

Master Equation:

$$\frac{d\rho}{dt} = \frac{1}{i\hbar}[H,\rho] + \frac{\kappa}{2}\sum_{j=-N}^{N}\left(2a_j\rho a_j^\dagger - a_j^\dagger a_j\rho - \rho a_j^\dagger a_j\right)$$

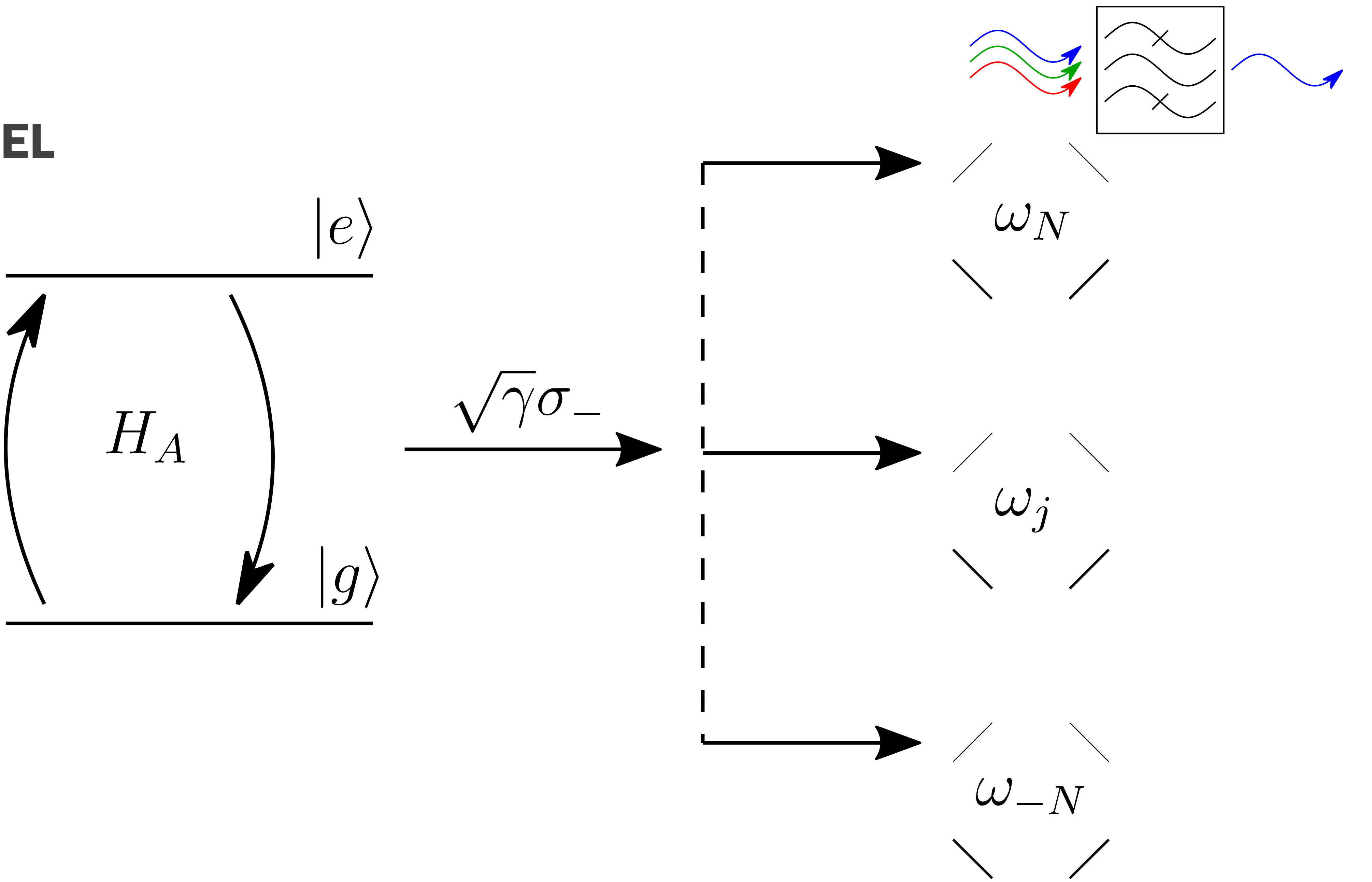
Filtered output

$$+ \frac{1}{2}\sum_{j=-N}^{N}\left(2C_j\rho C_j^\dagger - C_j^\dagger C_j\rho - \rho C_j^\dagger C_j\right)$$

Cascaded system coupling

$$C_j = \sqrt{\frac{\gamma}{2N+1}}e^{imj\pi/N}\sigma_- + \sqrt{\kappa}a_j$$

OUR MODEL



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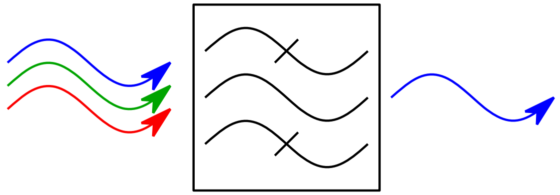
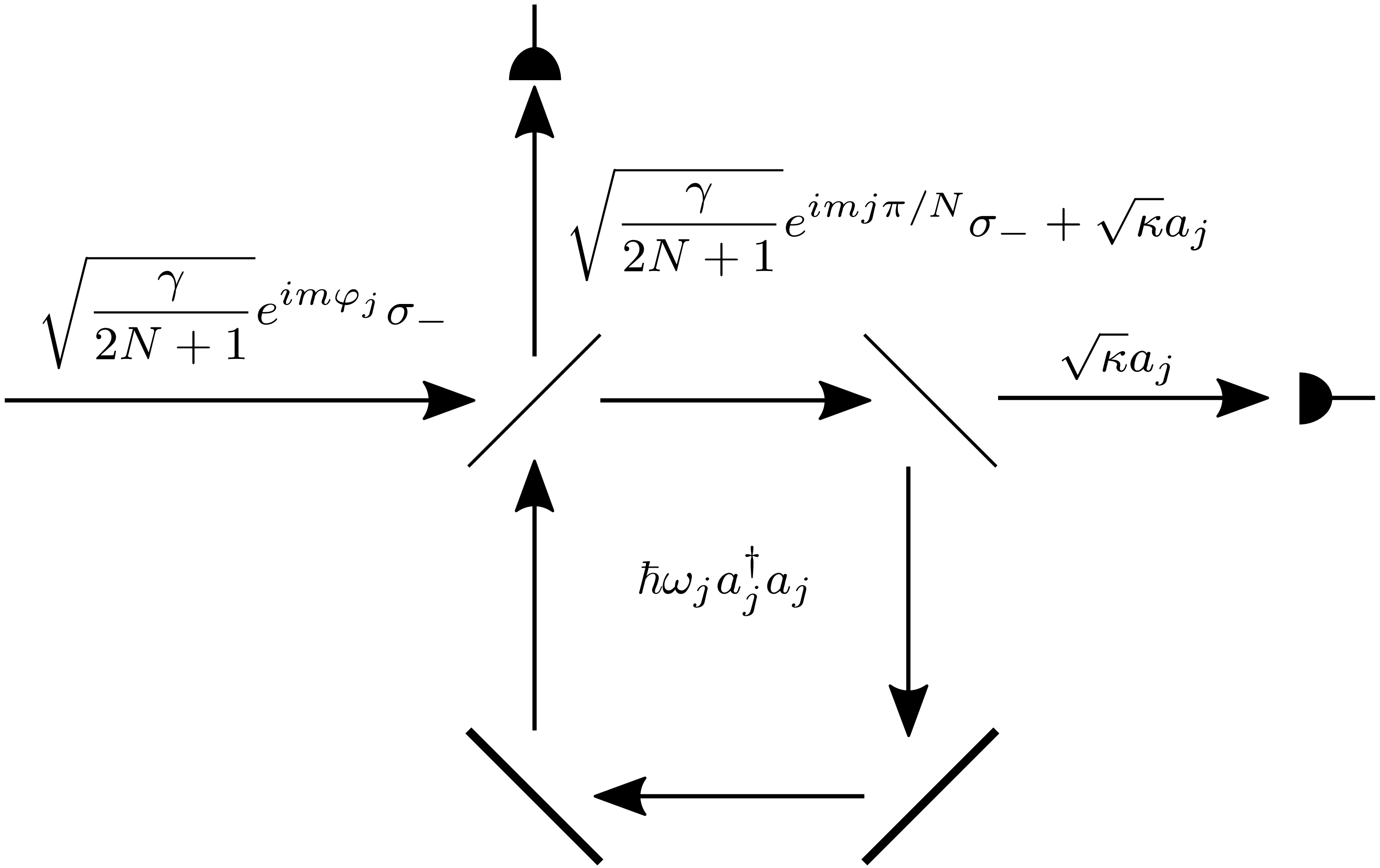
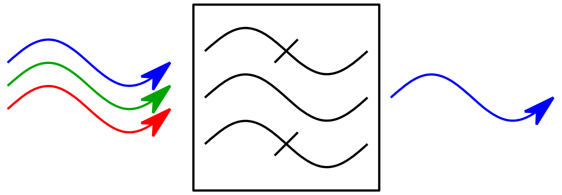
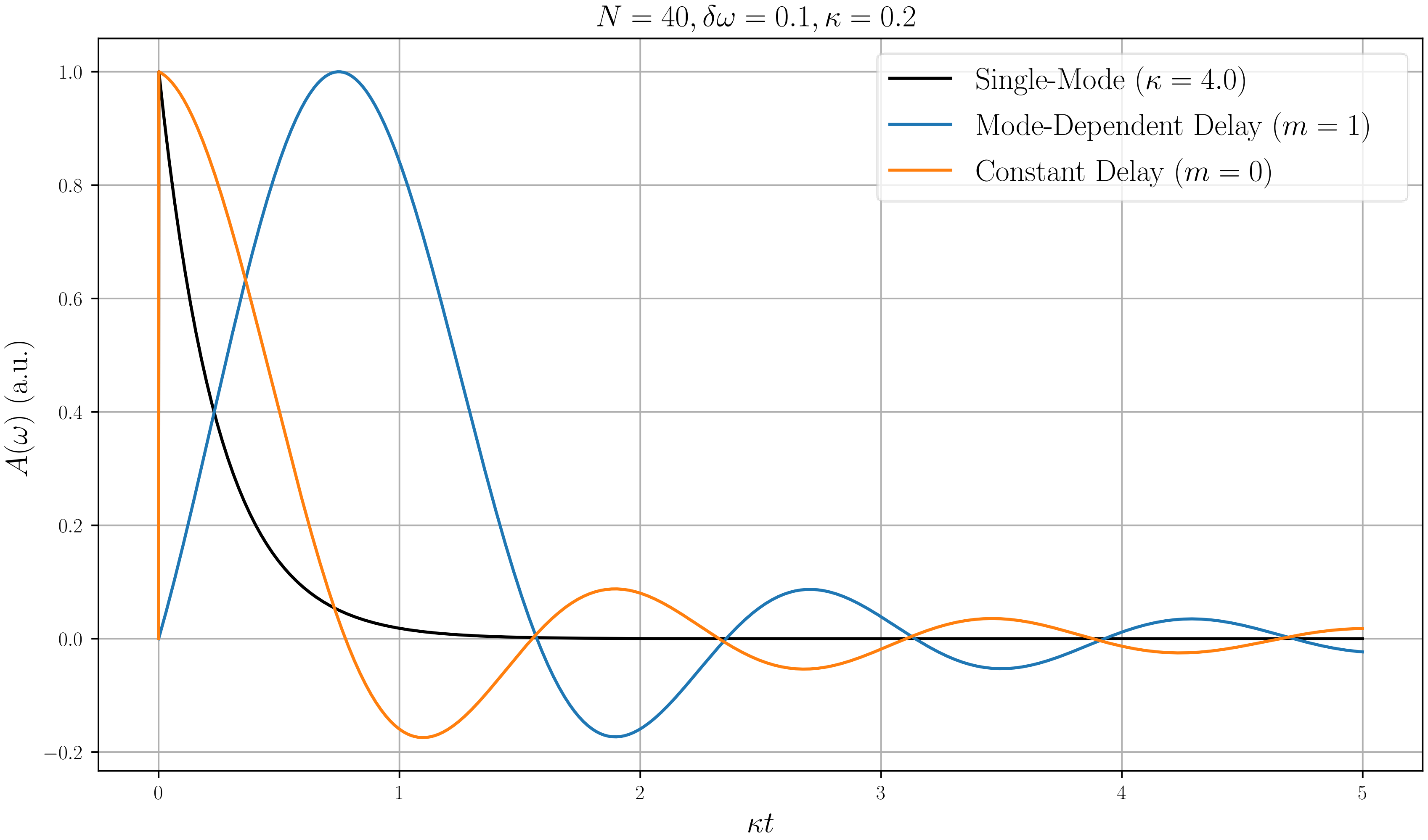


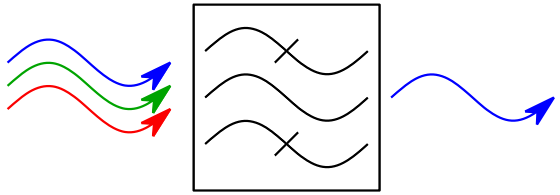


$\sqrt{\frac{\gamma}{2N+1}}e^{im\varphi_j}\sigma_-$
$\sqrt{\frac{\gamma}{2N+1}}e^{imj\pi/N}\sigma_- + \sqrt{\kappa}a_j$
$\sqrt{\kappa}a_j$
$\hbar\omega_j a_j^\dagger a_j$



CAVITY IMPULSE RESPONSE





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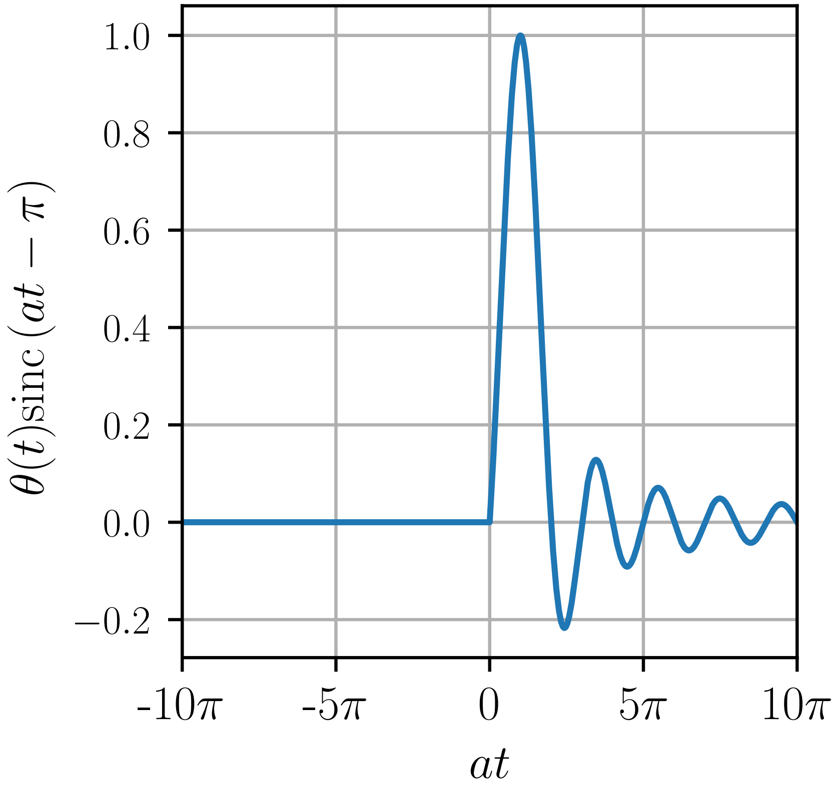
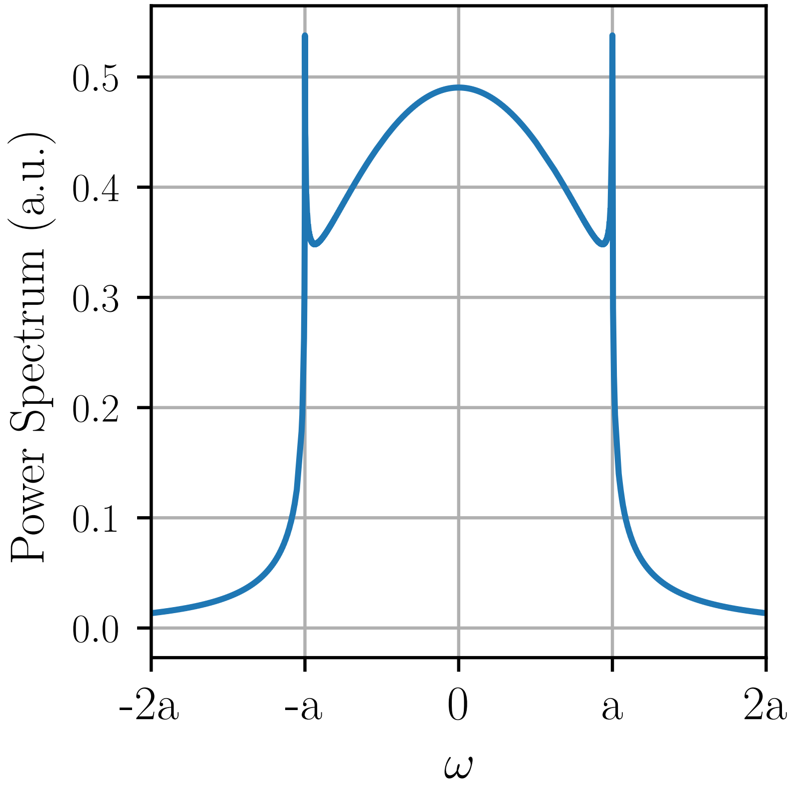
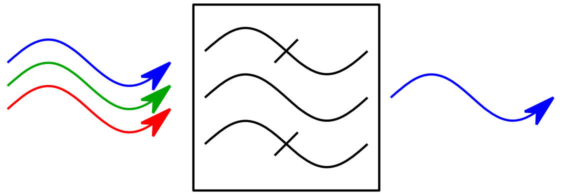
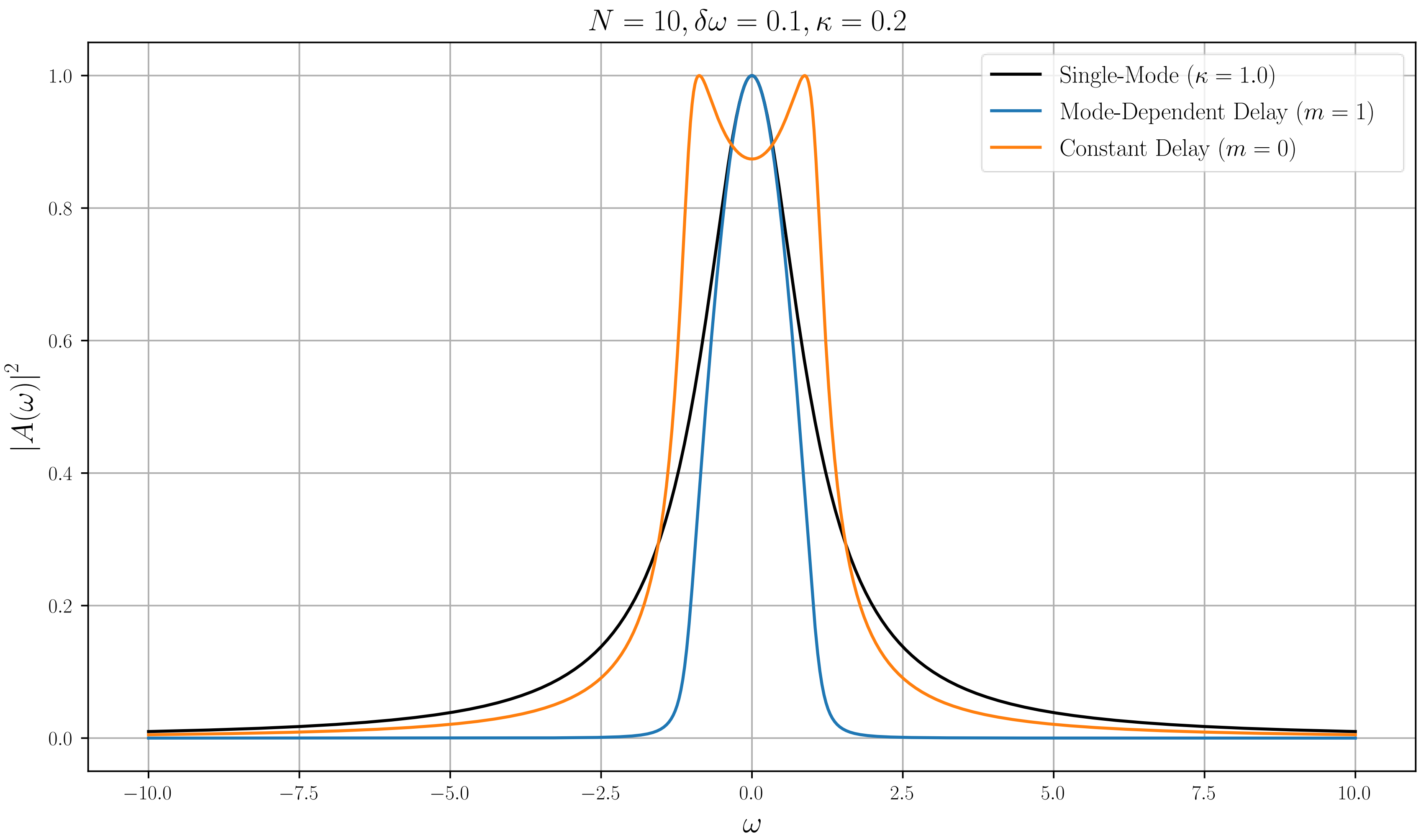






CAVITY FREQUENCY RESPONSE



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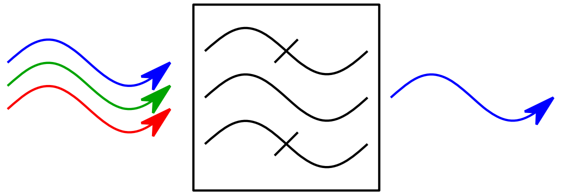

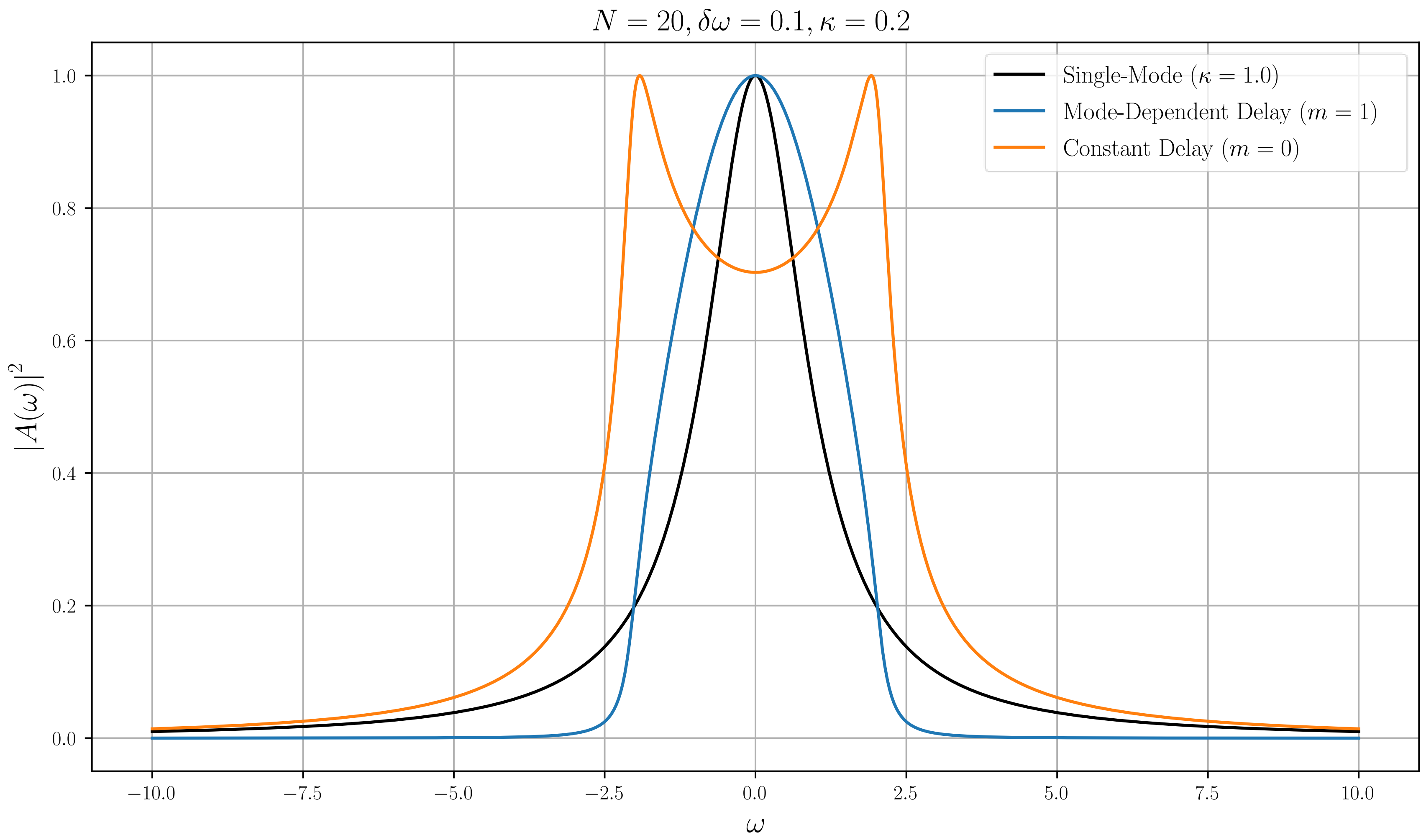

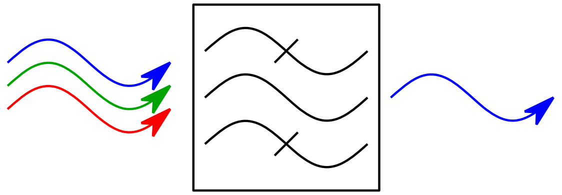

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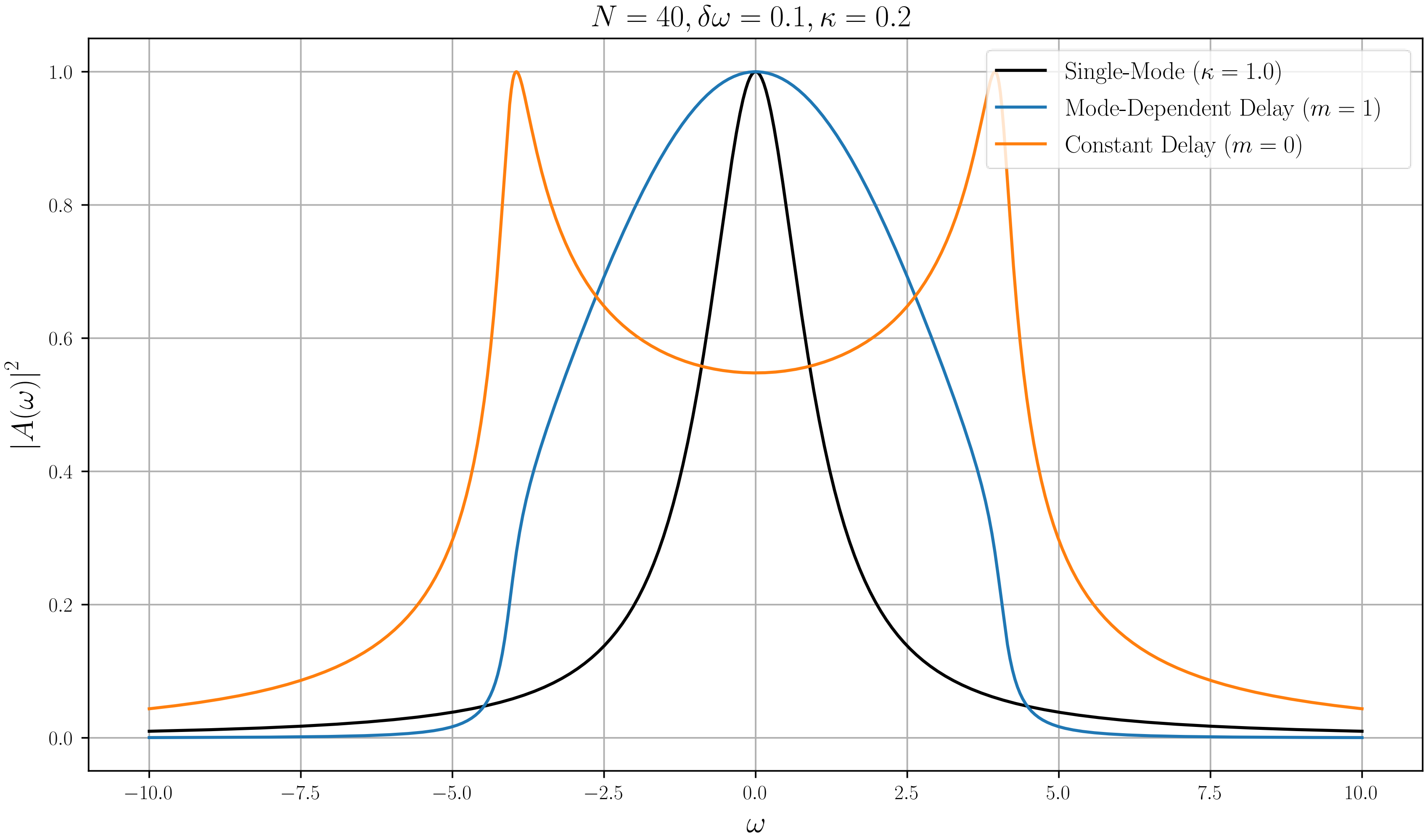
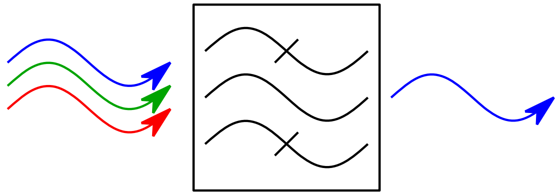




FILTERED POWER SPECTRUM

- Filtered spectrum is Fourier transform of first-order correlation function

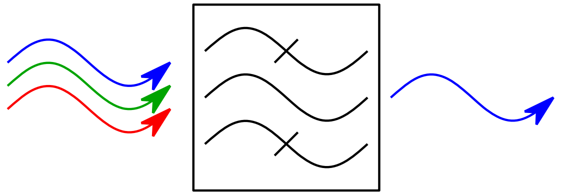
$$g^{(1)}(\tau) = \frac{\langle A^{\dagger}(\tau)A(0)\rangle}{\langle A^{\dagger}A\rangle_{ss}}$$

- Output spectrum of filter should be a single Lorentzian

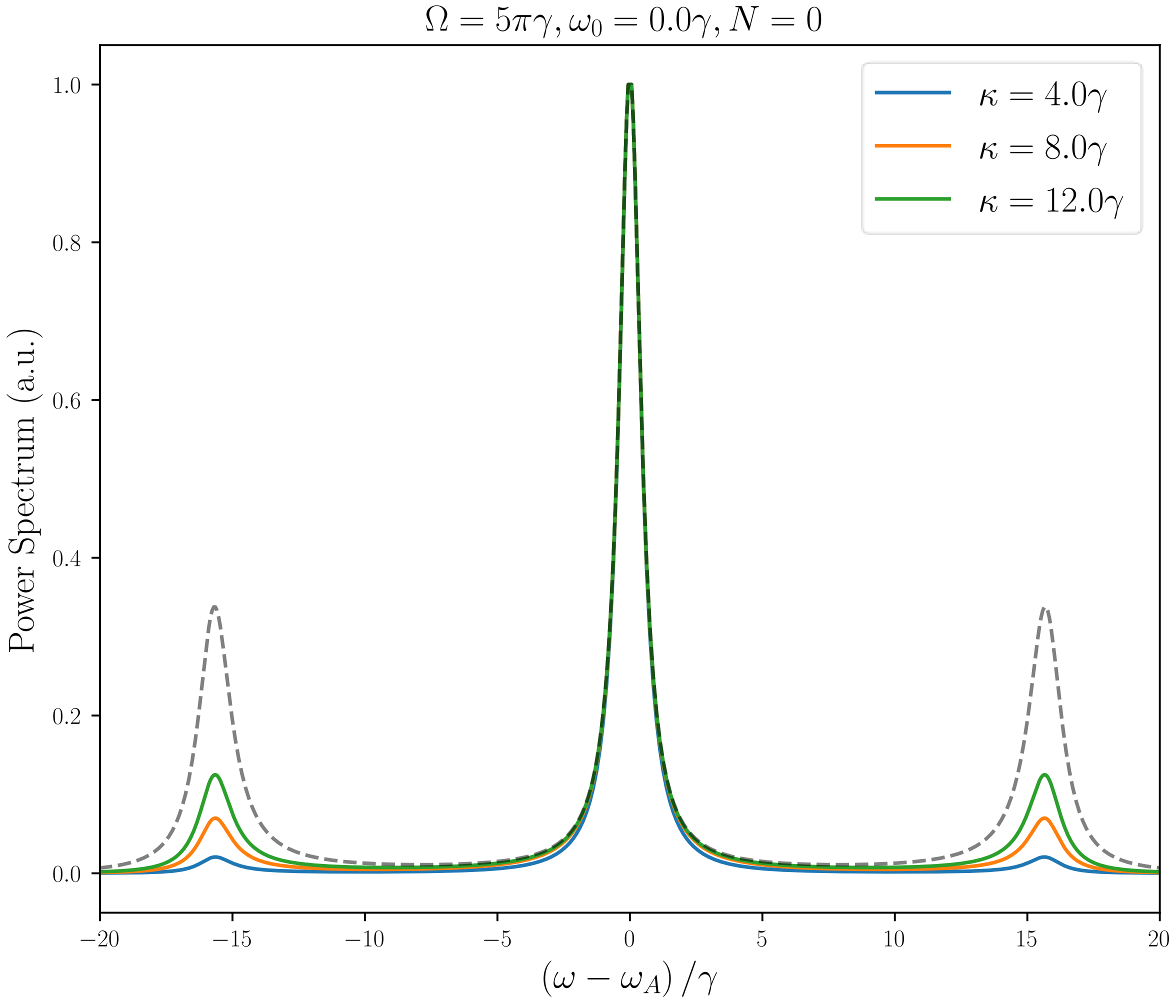
- Filtered photon correlation

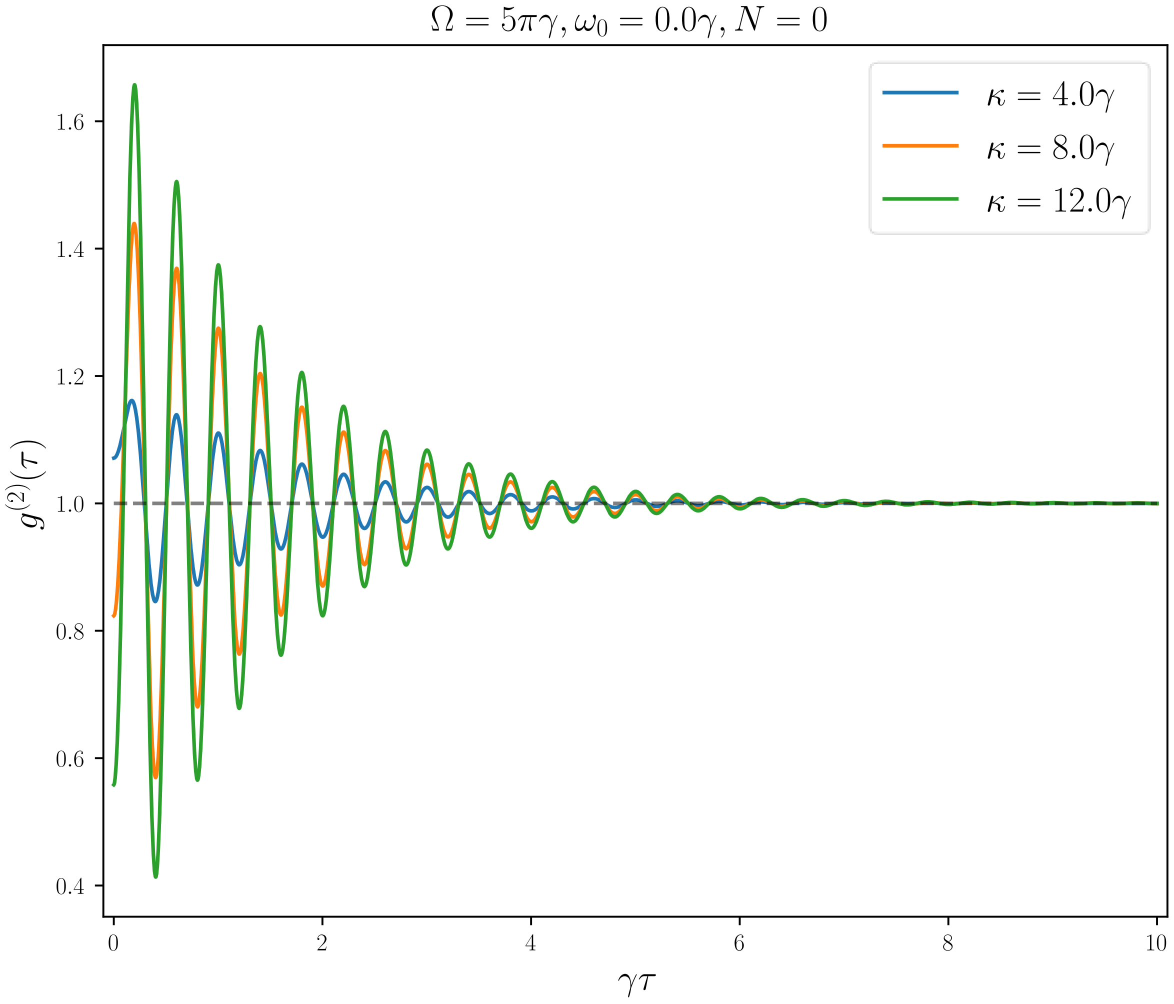
$$g^{(2)}(\tau) = \frac{\langle A^{\dagger}(0)A^{\dagger}(\tau)A(\tau)A(0)\rangle}{\langle A^{\dagger}A\rangle_{ss}^{2}}$$

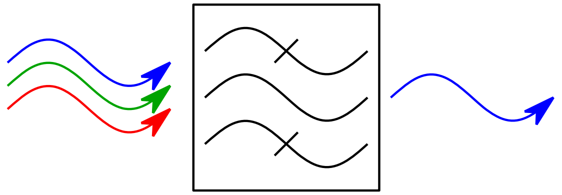
- Correlating photons from dressed state transitions



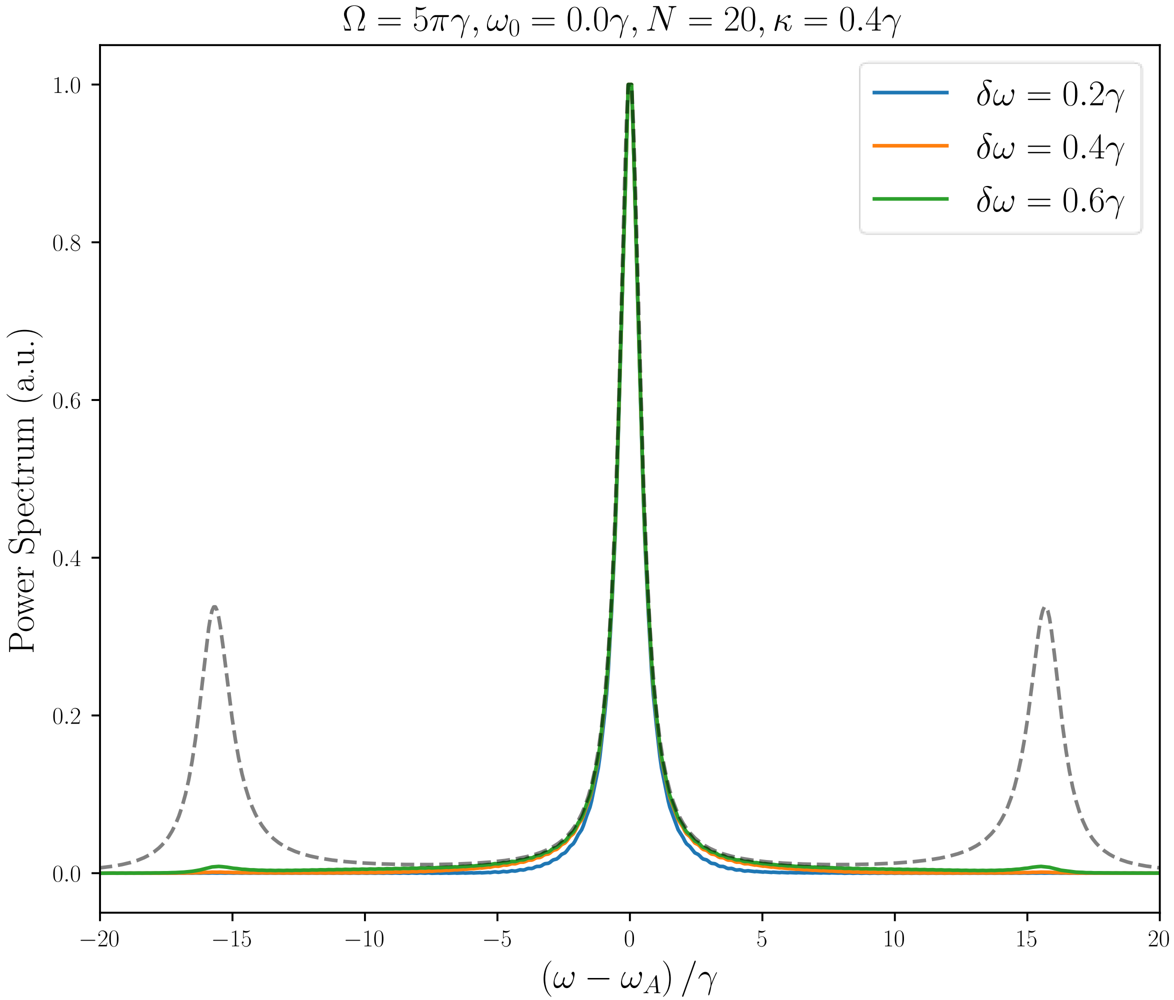
CENTRAL PEAK: SINGLE-MODE

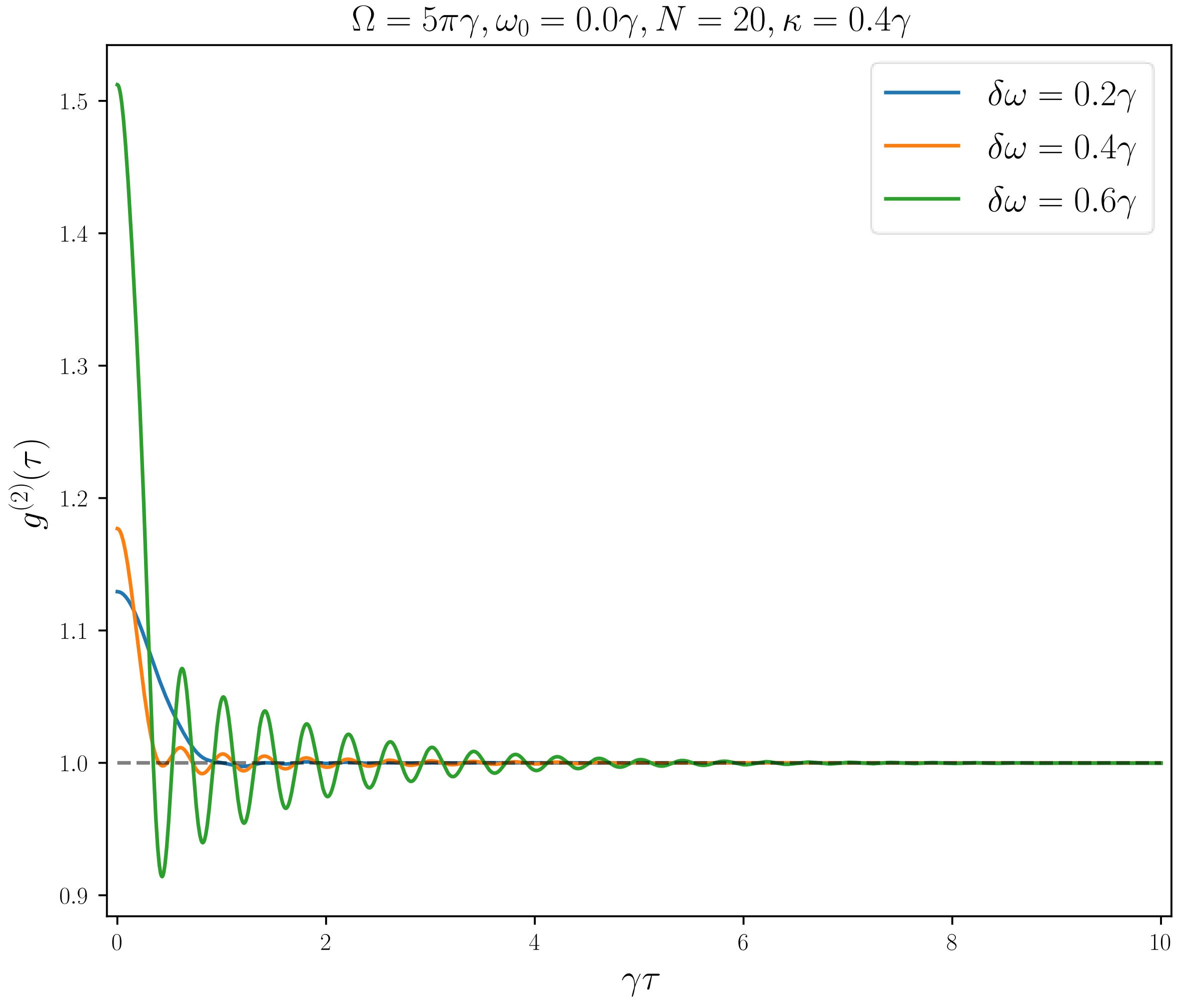


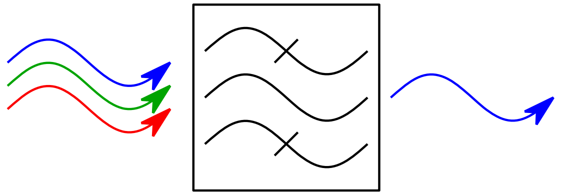




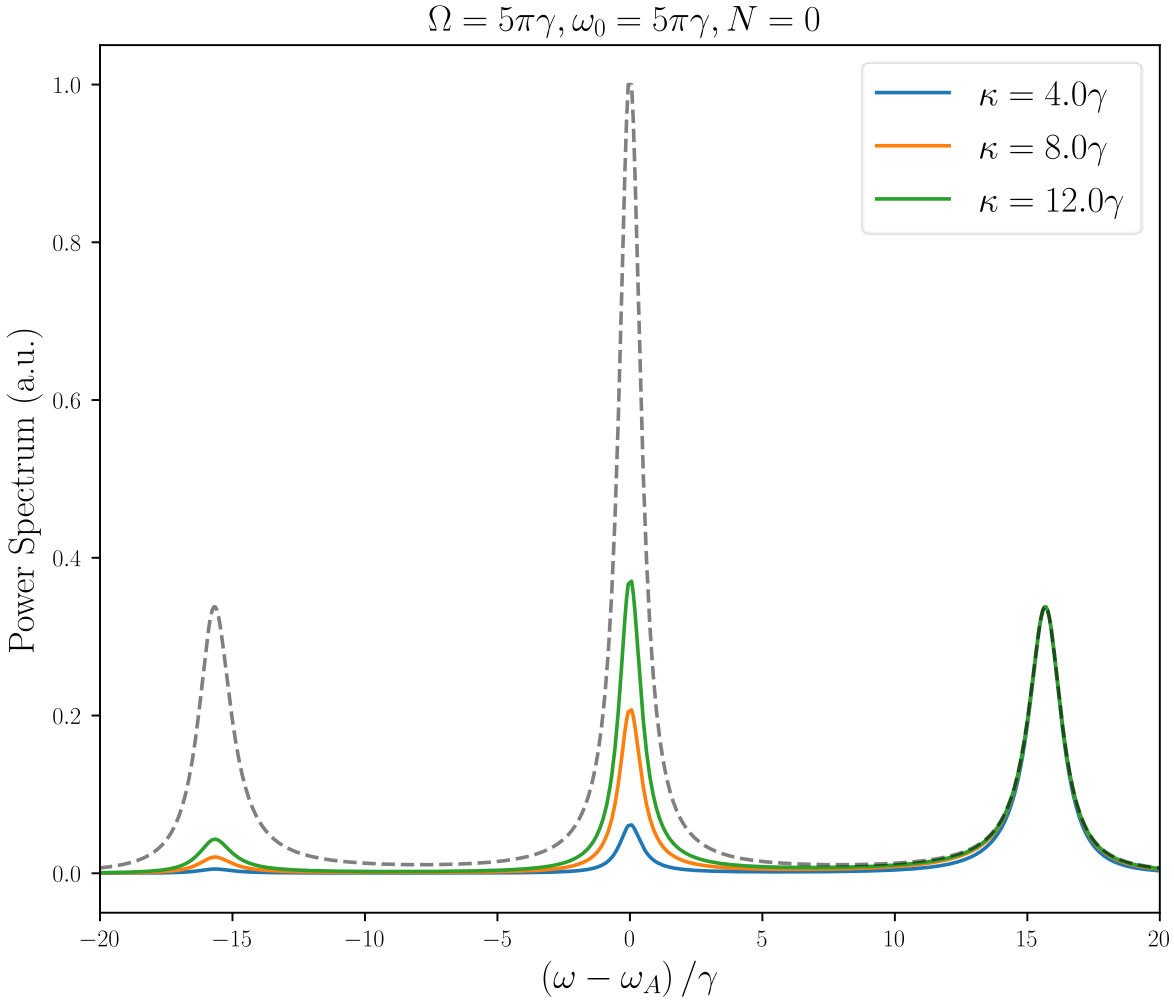
CENTRAL PEAK: MULTI-MODE

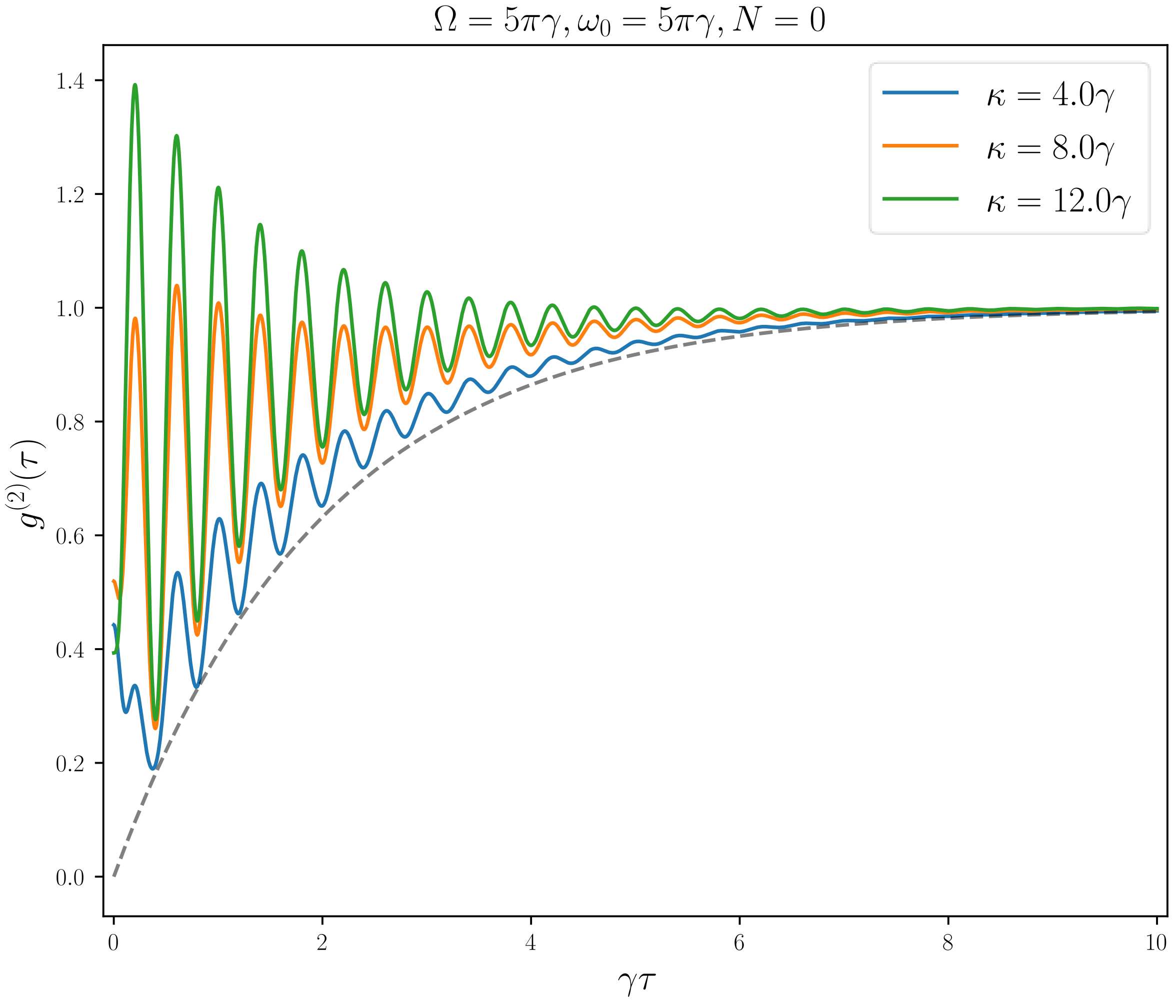


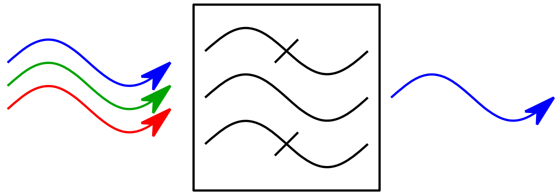




RIGHT SIDE-PEAK: SINGLE-MODE







RIGHT SIDE-PEAK: MULTI-MODE