A Better Method for Calculating Filtered Photon Correlations

Jacob P. K. Ngaha[1,2], and H. J. Carmichael[1,2]

[1]Department of Physics, The University of Auckland, Private Bag 92019, Auckland, New Zealand

[2]The Dodd-Walls Centre for Photonic and Quantum Technologies

Dodd-Walls Centre for Photonic and Quantum Technologies

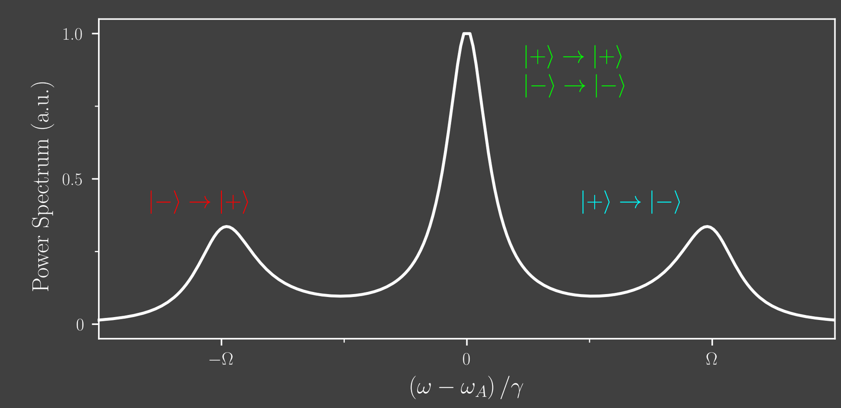
The University of Auckland

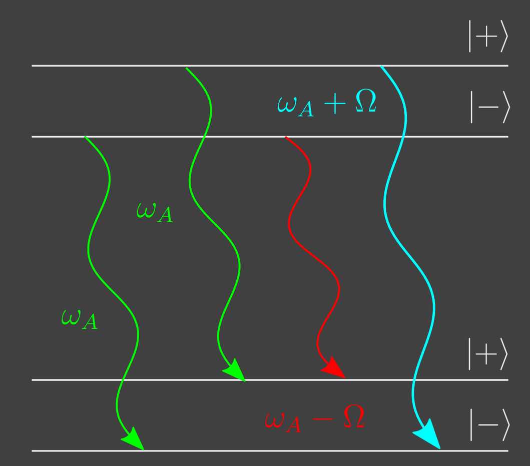
Quantum correlations for filtered fluorescence is a field with a rich history and, with the advent of quantum dots, continues to be studied [1-3]. In this work we develop an efficient theoretical approach to better filter fluorescence from a driven system. To demonstrate this, we model a resonantly driven two-level system coupled as a cascaded system into a phased array of tunable single-mode filter cavities.

We model the two-level system with Hamiltonian and master equation in the interaction picture

$$H_A = \hbar\frac{\Omega}{2}(\sigma_+ + \sigma_-), \qquad \frac{d\rho}{dt} = \frac{1}{i\hbar}[H,\rho] + \frac{\gamma}{2}(2\sigma_+\rho\sigma_- - \sigma_+\sigma_-\rho - \rho\sigma_+\sigma_-),$$

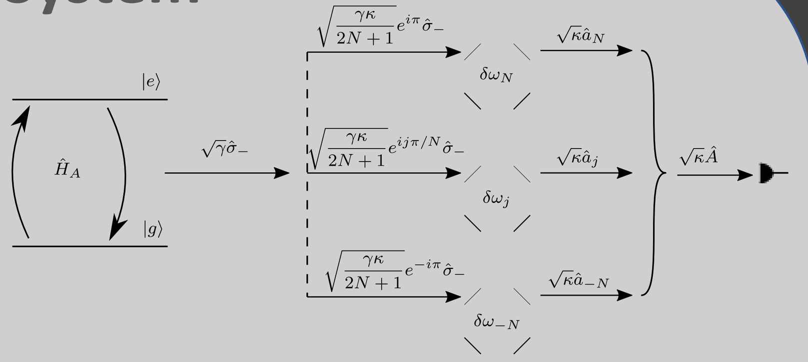
where Ω is the Rabi oscillation, γ is the atomic decay rate, and $\sigma_+(\sigma_-)$ is the atomic raising (lowering) operator. We can diagonalise the Hamiltonian to find the dressed states and dressed state quasi-energies $\pm\hbar\frac{\Omega}{2}$. In the dressed state picture, we see different transitions give rise to different peaks in the power spectrum of the fluorescence.





The System

We model the entire system as a cascaded open system, where the fluorescence from the two-level atom is equally coupled into a phased array of tunable single-mode cavities. The cascaded system master equation is:



$$\frac{d\rho}{dt} = -i\frac{\Omega}{2}(\sigma_+\rho - \rho\sigma_+) - i\frac{\Omega}{2}(\sigma_-\rho - \rho\sigma_-) + \frac{\gamma}{2}(2\sigma_-\rho\sigma_+ - \sigma_+\sigma_-\rho - \rho\sigma_+\sigma_-)$$
$$-i\sum_{j=-N}^{N}\delta\omega_j\left(a_j^\dagger a_j\rho - \rho a_j^\dagger a_j\right) + \kappa\sum_{j=-N}^{N}\left(2a_j\rho a_j^\dagger - a_j^\dagger a_j\rho - \rho a_j^\dagger a_j\right)$$
$$-\sqrt{\frac{\gamma\kappa}{2N+1}}\sum_{j=-N}^{N}e^{\frac{ij\pi}{N}}\left(a_j^\dagger\sigma_-\rho - \sigma_-\rho a_j^\dagger\right) - \sqrt{\frac{\gamma\kappa}{2N+1}}\sum_{j=-N}^{N}e^{\frac{-ij\pi}{N}}\left(a_j\sigma_+\rho - \sigma_+\rho a_j\right)$$

We assume each individual mode has a small bandwidth κ compared with the atomic linewidth γ. The filter is centred on a central frequency detuned from the atom frequency, $\delta\omega_0$, with individual mode spacing $\delta\omega$ ($\delta\omega_j = \delta\omega_0 + j\delta\omega$). This gives an effective halfwidth of the multi-mode filter array $N\delta\omega$. We apply a mode-dependent phase modulation, $e^{ij\pi/N}$, to the driving of mode j, giving our filter a box-like frequency response.

Operator Moment Equations

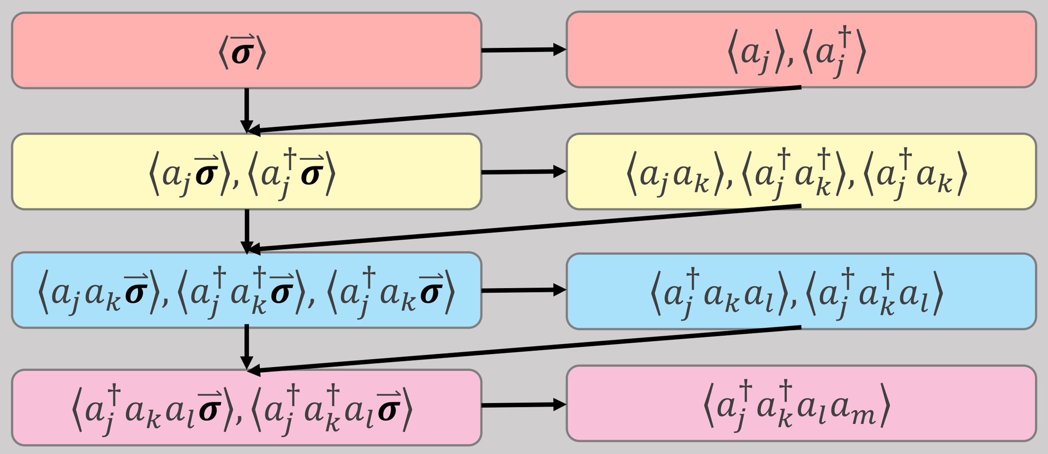
We could solve the master equation in matrix form, however as the number of filter modes increases the matrix becomes exceedingly large. We can instead calculate operator moment equations.
Due to the cascaded coupling of the fluorescence into the cavity modes, these equations also follow a cascaded pattern, where, beginning with the first order equations,

$$\frac{d}{dt}\langle\vec{\boldsymbol{\sigma}}(t)\rangle = \begin{pmatrix} -\frac{\gamma}{2} & 0 & i\frac{\Omega}{2} \\ 0 & -\frac{\gamma}{2} & -i\frac{\Omega}{2} \\ i\Omega & -i\Omega & \gamma \end{pmatrix}\langle\vec{\boldsymbol{\sigma}}(t)\rangle + \begin{pmatrix} 0 \\ 0 \\ -\gamma \end{pmatrix}, \qquad \langle\vec{\boldsymbol{\sigma}}(t)\rangle = \begin{pmatrix} \langle\sigma_-(t)\rangle \\ \langle\sigma_+(t)\rangle \\ \langle\sigma_z(t)\rangle \end{pmatrix},$$

and

$$\frac{d}{dt}\langle a_j(t)\rangle = -(\kappa + i\,\omega_j)\langle a_j(t)\rangle - \sqrt{\frac{\gamma\kappa}{2N+1}}e^{\frac{ij\pi}{N}}\langle\sigma_-(t)\rangle,$$

solutions to lower-order equations feed into higher order equations, with no coupling in the reverse direction.



Computing Correlation Functions

To compute the filtered photon correlations, we calculate the second-order correlation function:

$$g^{(2)}_{\text{Auto}} = \frac{\langle A^\dagger(t)A^\dagger A(t+\tau)A(t)\rangle}{\langle A^\dagger A(t)\rangle^2}, \qquad g^{(2)}_{\text{Cross}} = \frac{\langle B^\dagger(t)A^\dagger A(t+\tau)B(t)\rangle}{\langle A^\dagger A(t)\rangle\langle B^\dagger B(t)\rangle},$$

where $A = \sum_{j=-N}^{N} a_j$ and $B = \sum_{j=-N}^{N} b_j$. We make use of quantum regression formulae, where for a linear set of moment equations,

$$\frac{d}{dt}\langle\vec{\boldsymbol{O}}(t)\rangle = \boldsymbol{M}\langle\vec{\boldsymbol{O}}(t)\rangle,$$

the two-time correlation function obeys the same evolution equation:

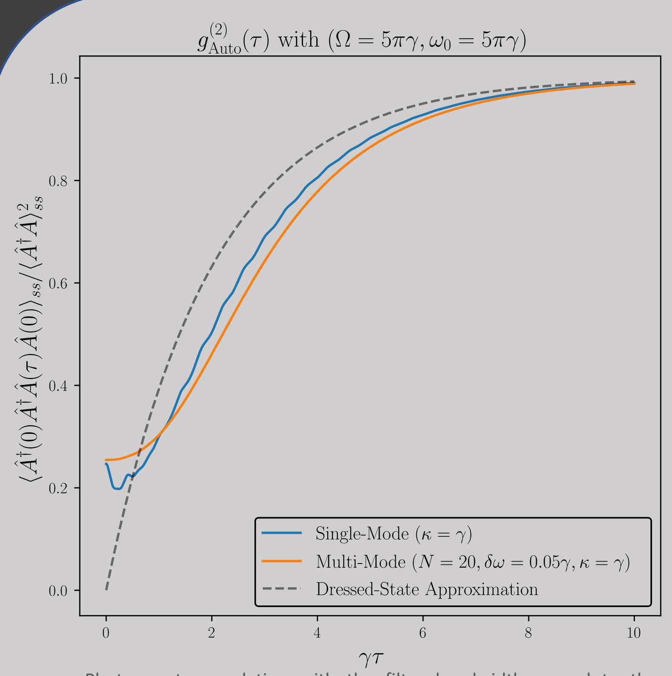
$$\frac{d}{d\tau}\langle X^\dagger(t)\vec{\boldsymbol{O}}(t+\tau)X(t)\rangle = \boldsymbol{M}\langle X^\dagger(t)\vec{\boldsymbol{O}}(t+\tau)X(t)\rangle,$$

where *X* is either *A* or *B*. The filtered photon correlations then follow from

$$\frac{d}{d\tau}\langle X^\dagger(0)a_j^\dagger a_k(\tau)X(0)\rangle = -[2\kappa - i(\omega_j - \omega_k)]\langle X^\dagger(0)a_j^\dagger a_k(\tau)X(0)\rangle$$
$$-\sqrt{\frac{\gamma\kappa}{2N+1}}e^{-\frac{ij\pi}{n}}\langle X^\dagger(0)\sigma_+ a_k(\tau)X(0)\rangle$$
$$-\sqrt{\frac{\gamma\kappa}{2N+1}}e^{\frac{ij\pi}{n}}\langle X^\dagger(0)\sigma_- a_k(\tau)X(0)\rangle,$$

and the equations up to second-order that govern the evolution of the driving terms on the right-hand side. Due to the cascaded structure of the equations, the computation is extremely efficient. For example, *N=40* (81 modes) takes ***less than a second*** of computation time, while solving in the matrix form and for *N=20* (41 modes), computations took around ***20 hours!***

Auto-Correlations



Photon auto-correlation with the filter bandwidth equal to the inverse correlation time.

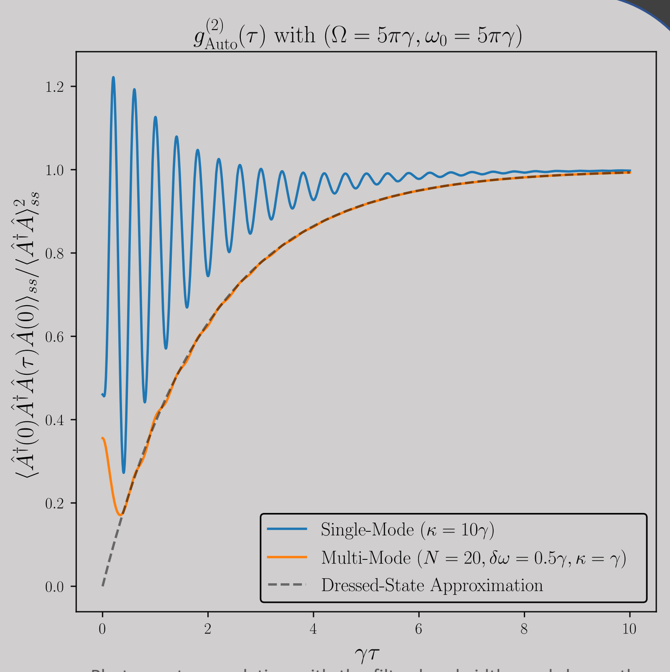
For photons of frequency $\omega_A \pm \Omega$, an ideal filter would capture the perfect *antibunched* auto-correlation, given by

$$g^{(2)}_{\text{Dressed}}(\tau) = 1 - e^{-\frac{\gamma}{2}\tau},$$

as depicted in the figures by the grey-dashed line.

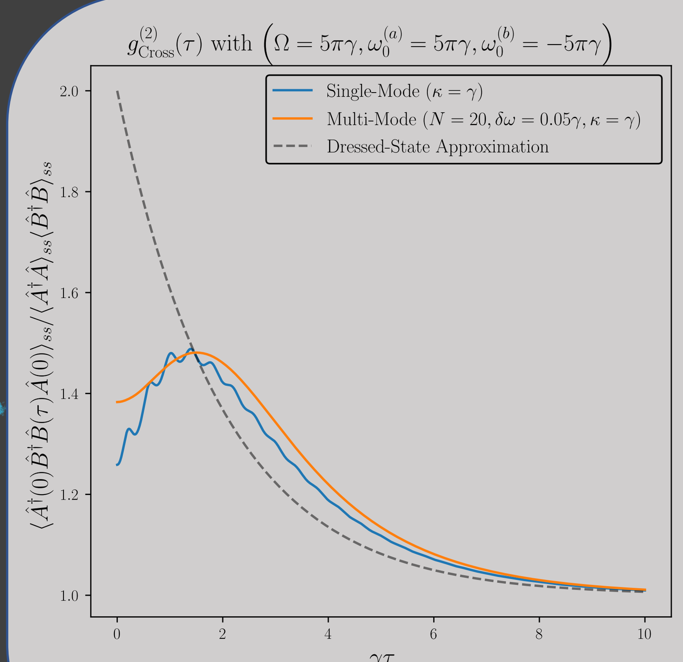
On the left, the filter bandwidth is too narrow to capture all of the dynamics, so neither the single-mode (blue) or multi-mode array (orange) perform well.

On the right the filter bandwidth is large enough to capture all of the dynamics. The single-mode filter now completely fails to isolate the single-frequency, with large Rabi oscillations visible. The multi-mode array almost perfectly recreates the dressed-state correlation.



Photon auto-correlation with the filter bandwidth much larger than the inverse correlation time.

Cross-Correlations



Photon cross-correlation of the right-peak conditioned on the left, with the filter bandwidth equal to the inverse correlation time.

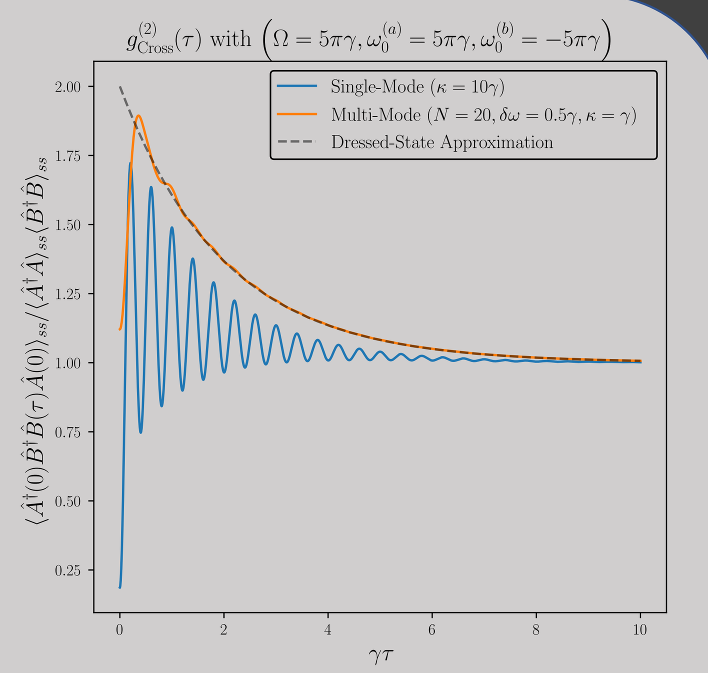
Due to the fast computation times and simple structure of the moment equations it is easy to implement a second tunable filter. Different frequency photons can then be cross-correlated and compared with the expected *bunched* dressed-state correlation

$$g^{(2)}_{\text{Dressed}}(\tau) = 1 + e^{-\frac{\gamma}{2}\tau}.$$

On the left, the filter bandwidth is much too narrow to capture all the dynamics of the transition. The single-mode filter still shows slight Rabi oscillations as it cannot isolate either frequency.

On the right the filter bandwidth is large enough to capture all of the dynamics. Again, the single-mode filter has failed to isolate either frequency resulting in large Rabi oscillations. The multi-mode filter demonstrates a much better frequency response for a larger bandwidth.

Solving the master equation in matrix form is too inefficient for these calculations.



Photon cross-correlation of the right-peak conditioned on the left, with the filter bandwidth much larger than the inverse correlation time.

References

[1] Aspect, A., Roger, G., Renaud, S., Dalibard, J. & Cohen-Tannoudji, C. *Phys. Rev. Lett.* **45**, 61—620 (1980) .

[2] Apanasevich, P. A., Kilin, S. Ya., & Federov, A. F. *J. Physics. B: At. Mol. Opt. Phys.* **18**, L55 (1985).

[3] Ulhaq, A., Weiler, S., Ulrich, M., Roßbach, R., & Jetter, M. *Nature Photonics*, 238 (2012).

[4] Carmichael, H. *Phys. Rev. Lett.* **70**, 2273 (1993).