PHASE RESETTING IN THE YAMADA MODEL OF A Q-SWITCHED LASER

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MOTIVATION – A WAY TO PROBE STABLE OSCILLATING SYSTEMS

- Optical frequency combs, optical clocks, and optical neural networks all work with stable oscillating optical systems.
- After an induced perturbation, the system will return back to stable oscillations.
- With phase resetting, we can study how the new phase of the return relates to the phase of the initial oscillation.

THE YAMADA MODEL

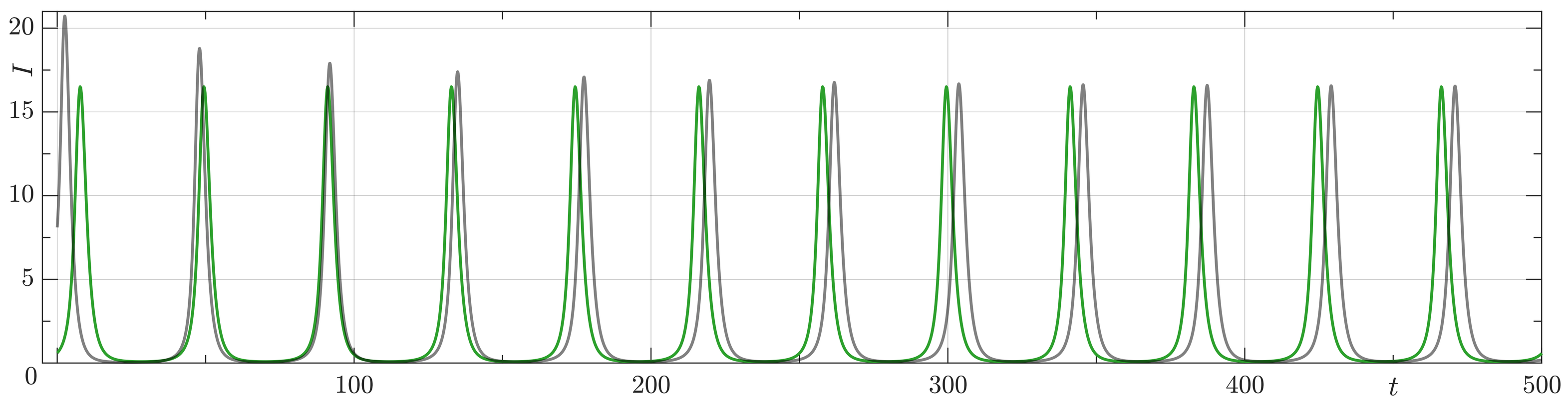
- Yamada model for a q-switched laser with a lossy saturable absorber:

$$\dot{G} = \gamma(A - G - GI)$$
$$\dot{Q} = \gamma(B - Q - aQI)$$
$$\dot{I} = I(G - Q - 1)$$

where:

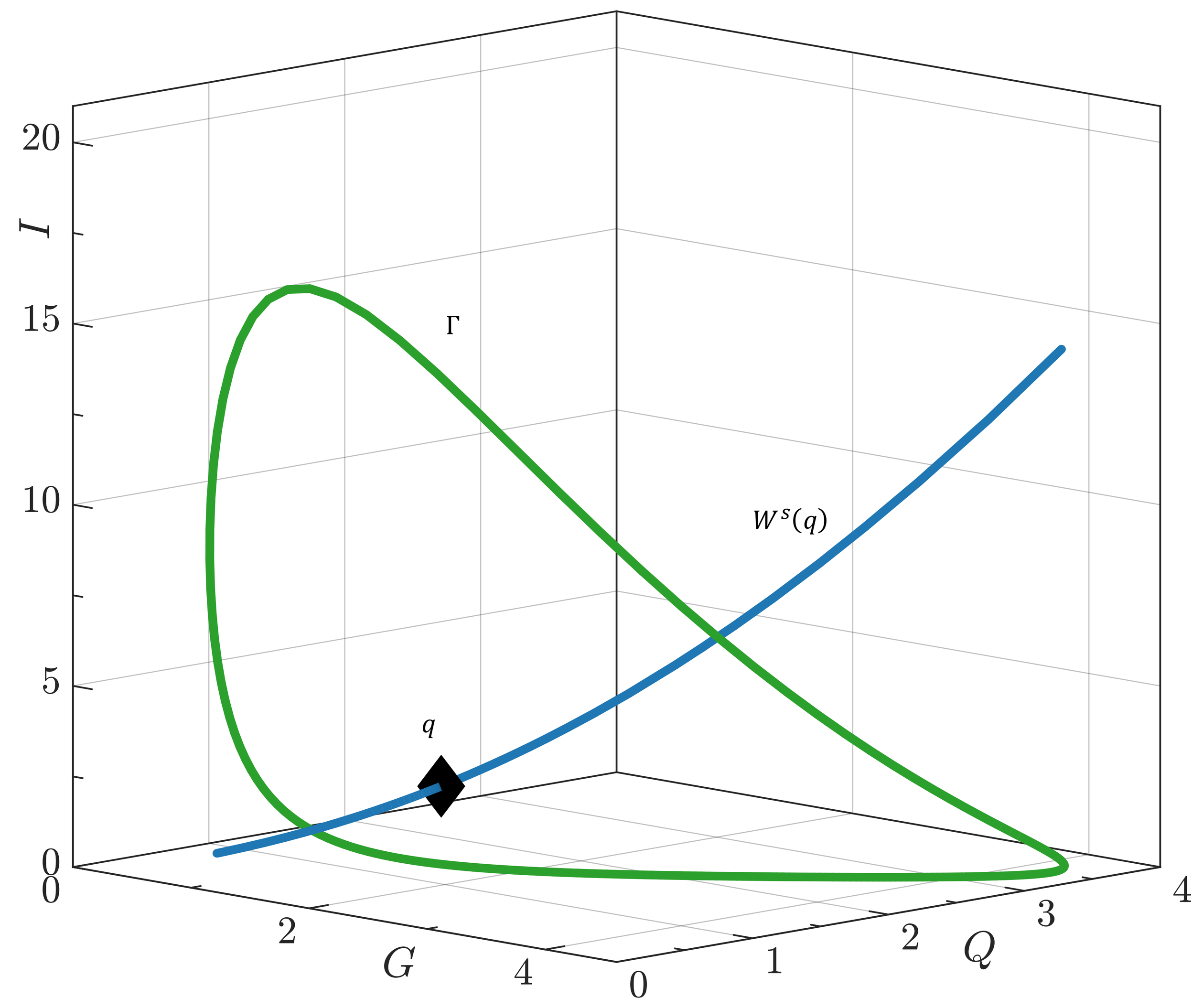
- G – gain,
- Q – absorption,
- I – laser intensity,
- $\gamma = 0.0354$ – photon loss rate,
- $A = 7.3757$ – pump current,
- $B = 5.8, a = 1.8$ – fixed, dimensionless parameters.

- Nine different regions of dynamics [1,2], separated by different bifurcations. Here we focus on a region with a stable periodic solution.



Time series of the unperturbed oscillations(green) and perturbed response (black) back to the original pulsations at t~300, with a phase shift.

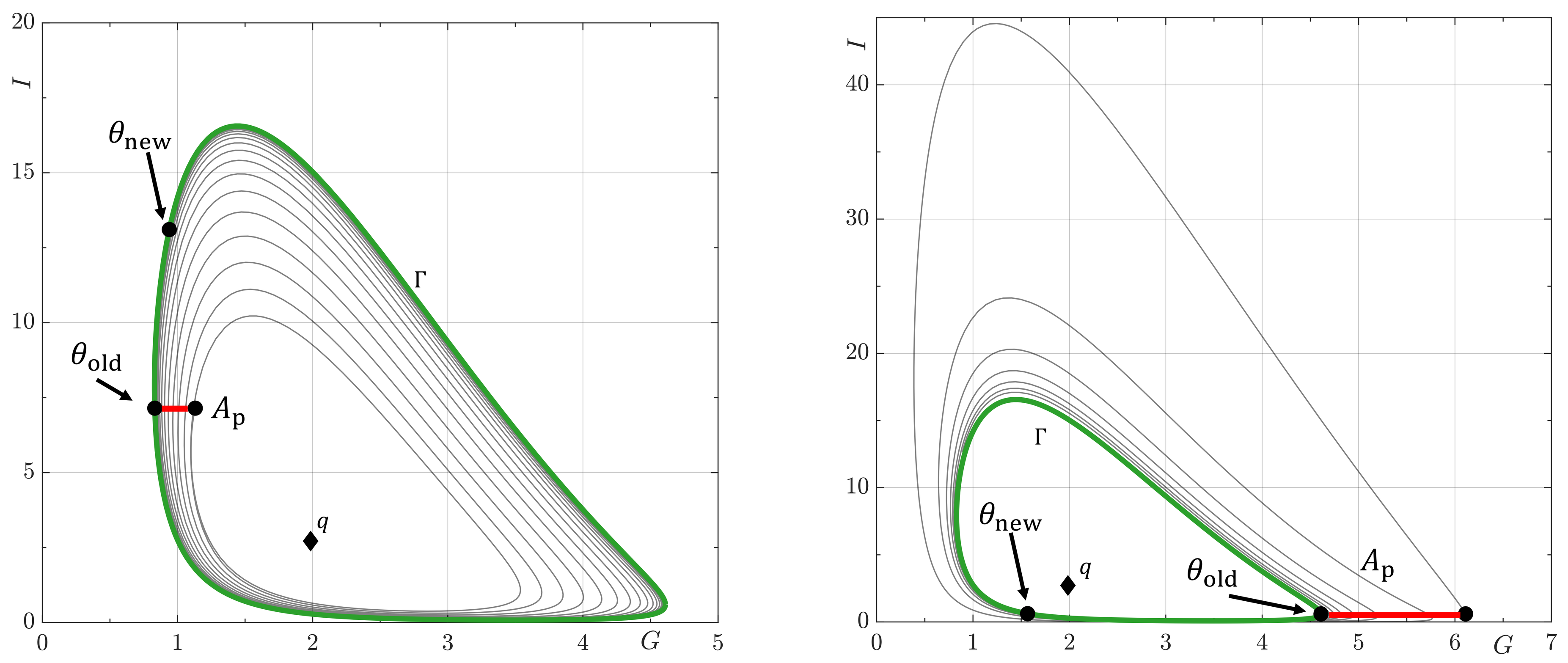
- Objects in phase space:
  - o – stable equilibrium (not shown),
  - q – unstable stationary point (diamond),
  - Γ – stable periodic orbit (green curve),
  - $W^s(q)$ - one-dimensional stable manifold of q (blue curve).
- All initial conditions not on $W^s(q)$ will evolve towards the periodic orbit Γ.



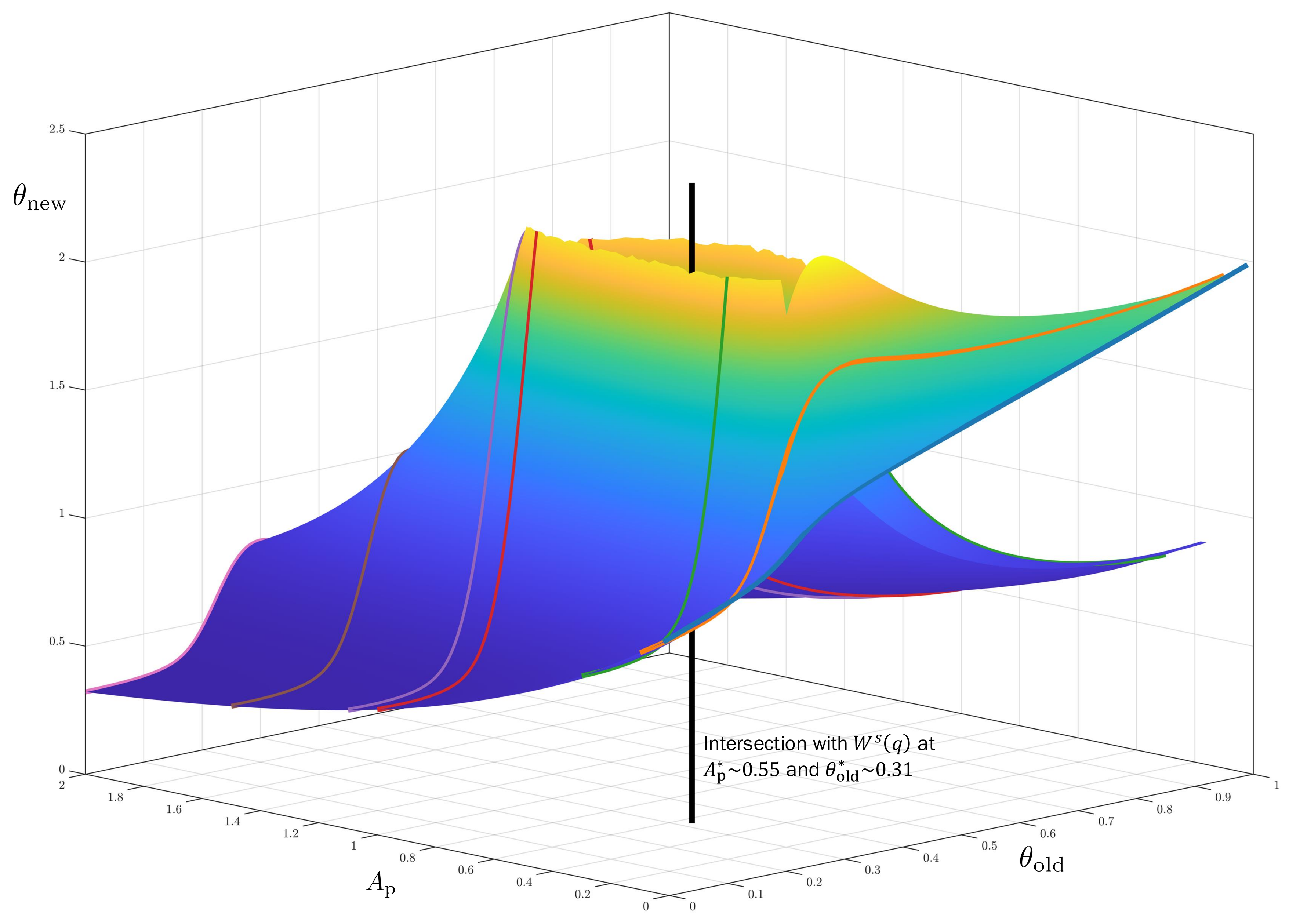
Three-dimensional phase portrait of the stable periodic orbit

PHASE RESETTING AND PHASE TRANSITION CURVES (PTCs)

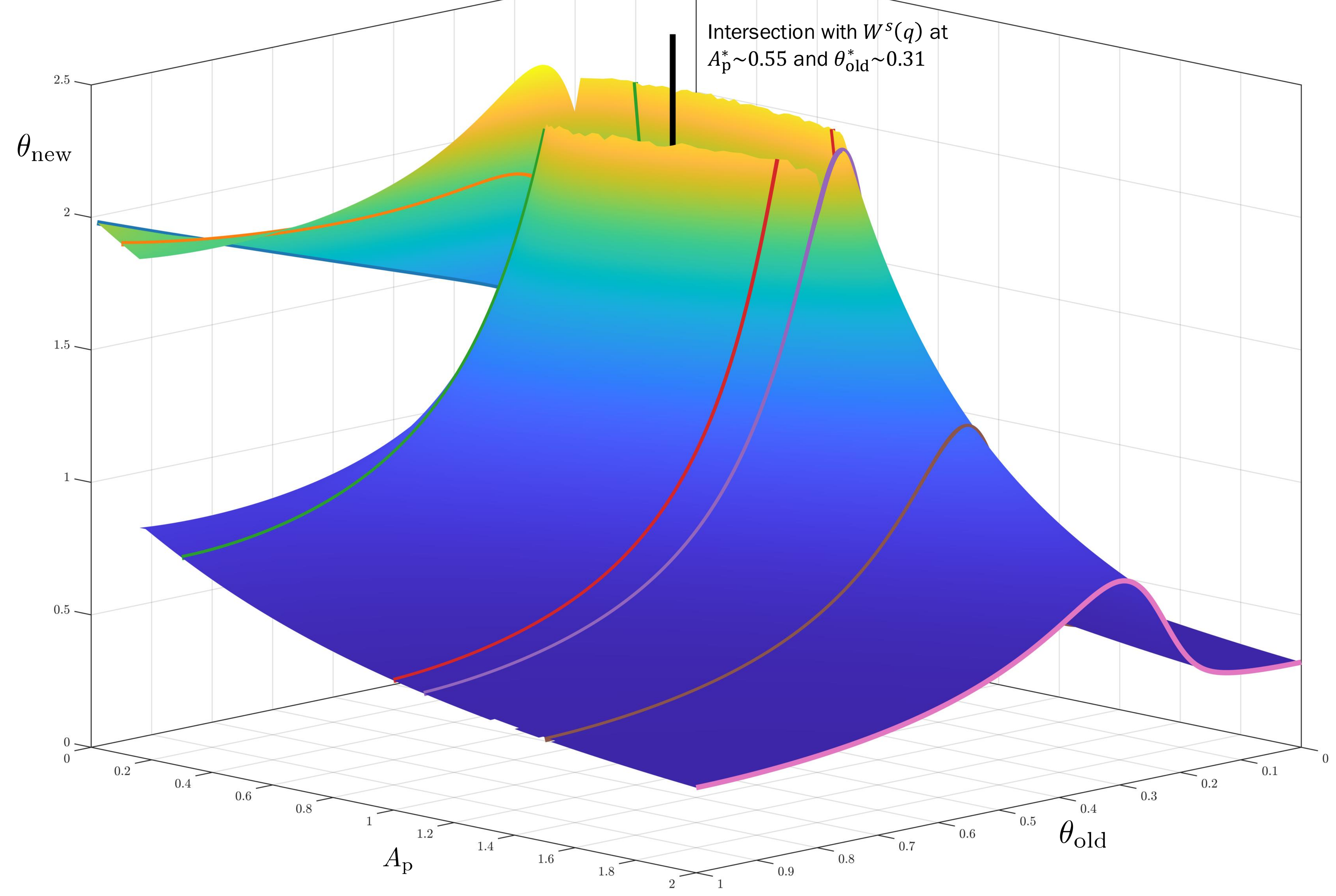
- Perturbations cause a phase shift ('lag') in intensity pulses [3,4].
- Phase resetting parameters for fixed perturbations in positive G-direction.
  - A_p – perturbation amplitude ,
  - θ_{old} – phase where perturbation is applied,
  - θ_{new} – phase where trajectory "resets".
- For weak perturbations, θ_{new} increases as θ_{old} increases.
- For strong perturbations, there is a topological change.
- For perturbation $A_p^* \sim 0.55$ at $\theta_{old}^* \sim 0.31$, Γ intersects $W^s(q)$. The trajectory does not return to Γ, and the phase is undefined.

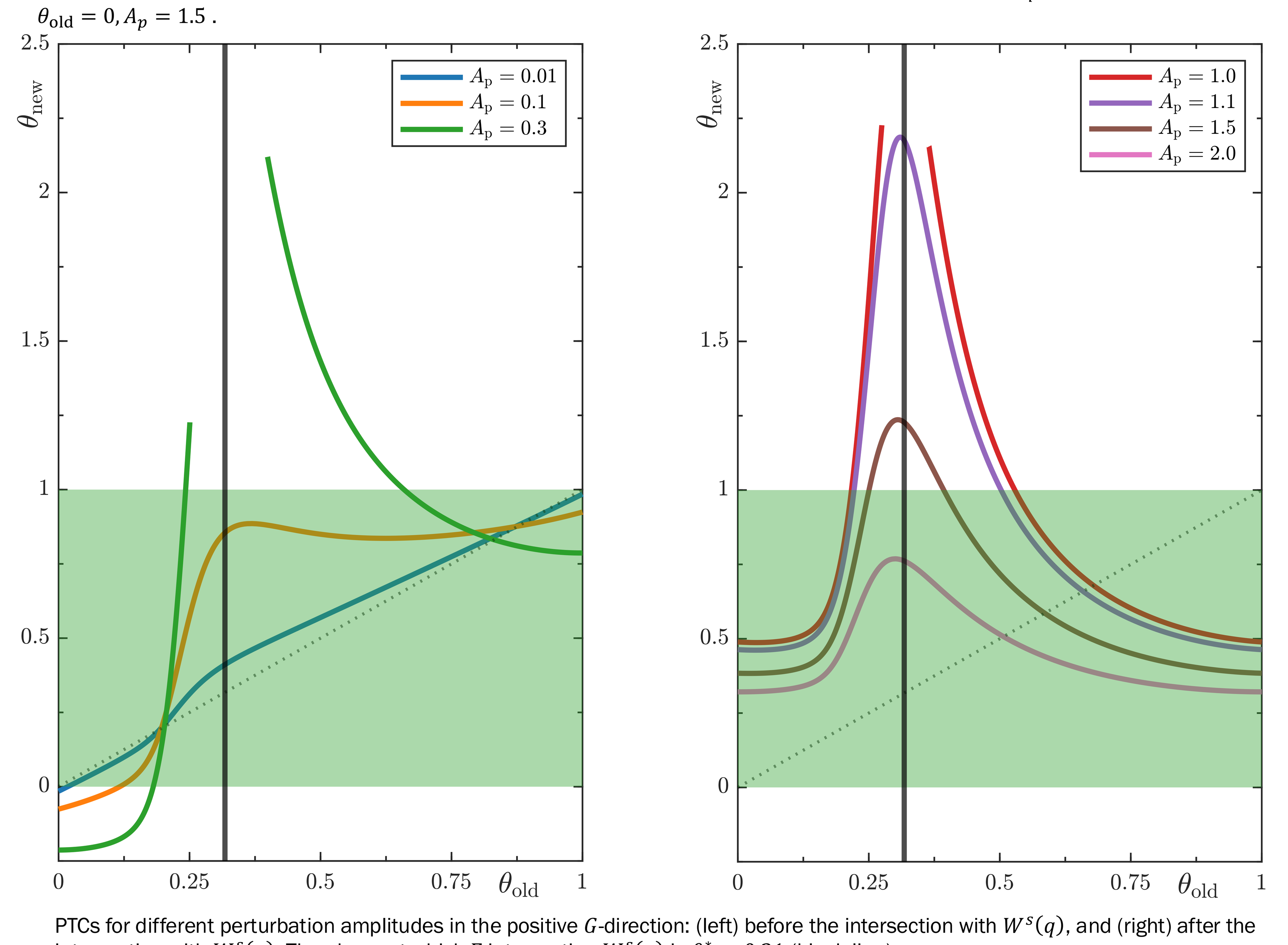


Phase portraits of the periodic orbit with a perturbation in the positive G-direction at: (left) $\theta_{old} = 0.5$ with $A_p = 0.3$; and at (right) $\theta_{old} = 0, A_p = 1.5$.



PTC surface for perturbations in the positive G-direction.





PTCs for different perturbation amplitudes in the positive G-direction: (left) before the intersection with $W^s(q)$, and (right) after the intersection with $W^s(q)$. The phase at which Γ intersection $W^s(q)$ is $\theta_{old}^* \sim 0.31$ (black line).

References

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[3] P. Langfield, B. Krauskopf, and H. M. Osinga. "A continuation approach to computing phase resetting curves" in *Advances in Dynamics, Optimization, and Computation"*, Vol 304 of *SSDC* (Springer International Publishing, 2020), pp. 3-30.
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