

# More Is More: Multi-Mode Filtered Photon Correlations



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## Abstract

We propose a novel method to investigate the nature of fluorescence from a driven atom. Using open systems theory we model a photon counting experiment whereby the fluorescence is coupled into a scanning interferometer (filtering cavity) [1-3]. We choose our cavity to be an array of different frequency modes each with a very fine Lorentzian shape, effectively modelling a bandpass or "box" type filter. The transmitted fluorescence is then detected by a photon counter, allowing us to calculate second-order correlations for the filtered field.

While the current presentation is concerned with a resonantly driven two-level atom, we aim to work with a three-level ladder type atom driven at two-photon resonance, as investigated by Gasparinetti et al. [4].

## Two-Level Dressed States

We have a driven two-level atom with Hamiltonian and master equation

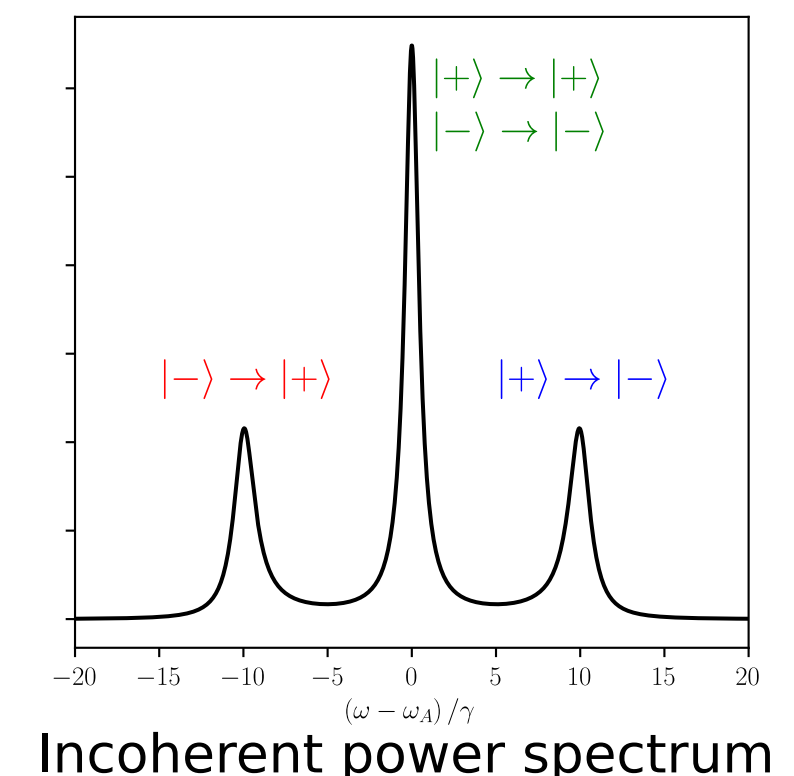
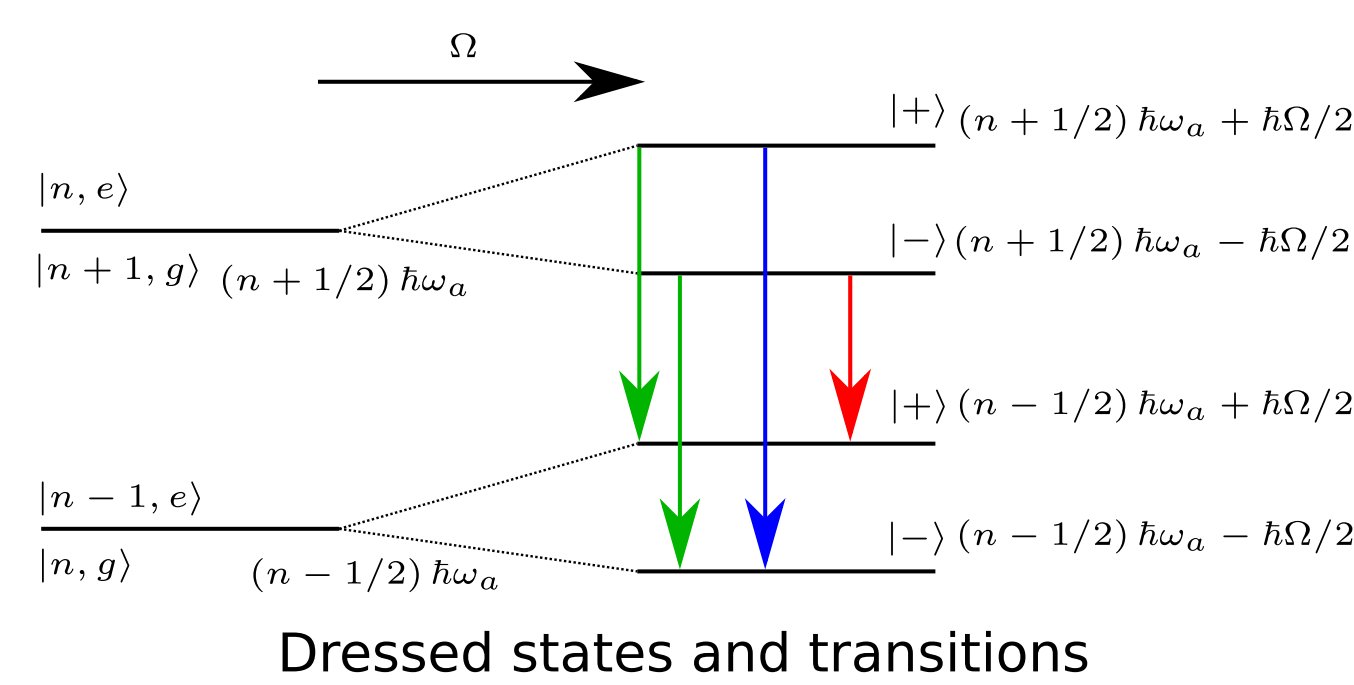
$$H_A = \hbar \frac{\Omega}{2} (\sigma_+ + \sigma_-) \quad \frac{d\rho}{dt} = \frac{1}{i\hbar} [H, \rho] + \frac{\gamma}{2} (2\sigma_- \rho \sigma_+ - \sigma_+ \sigma_- \rho - \rho \sigma_+ \sigma_-)$$

where  $\Omega$  is the driving strength (Rabi frequency),  $\sigma_- (\sigma_+) = |g\rangle \langle e| (|e\rangle \langle g|)$  is the atomic lowering (raising) operator, and  $\gamma$  is the atomic decay rate. We diagonalise the Hamiltonian to find the dressed states and frequencies

$$H_A |\pm\rangle = \pm \hbar \frac{\Omega}{2} |\pm\rangle \quad |\pm\rangle = \frac{1}{\sqrt{2}} (|g\rangle \pm |e\rangle)$$

In this dressed state picture, we see different transitions give rise to different peaks in the power spectrum of the fluorescence. We may take a secular approximation by considering the atomic operators in terms of the dressed states, allowing us to find analytic forms of the second-order correlation function for the different dressed state transitions:

$$g_{|\pm\rangle \rightarrow |\mp\rangle}^{(2)}(\tau) = 1 - e^{-\gamma/2\tau} \quad g_{|\pm\rangle \rightarrow |\pm\rangle}^{(2)}(\tau) = 1$$



## The Model

We couple the fluorescence to an array of cavity modes, which we describe with the Hamiltonian

$$H = H_A + \hbar \sum_{j=-N}^N \Delta_j a_j^\dagger a_j + \frac{i\hbar}{2} \sqrt{\gamma\kappa} (A\sigma_- - A^\dagger\sigma_+)$$

with master equation

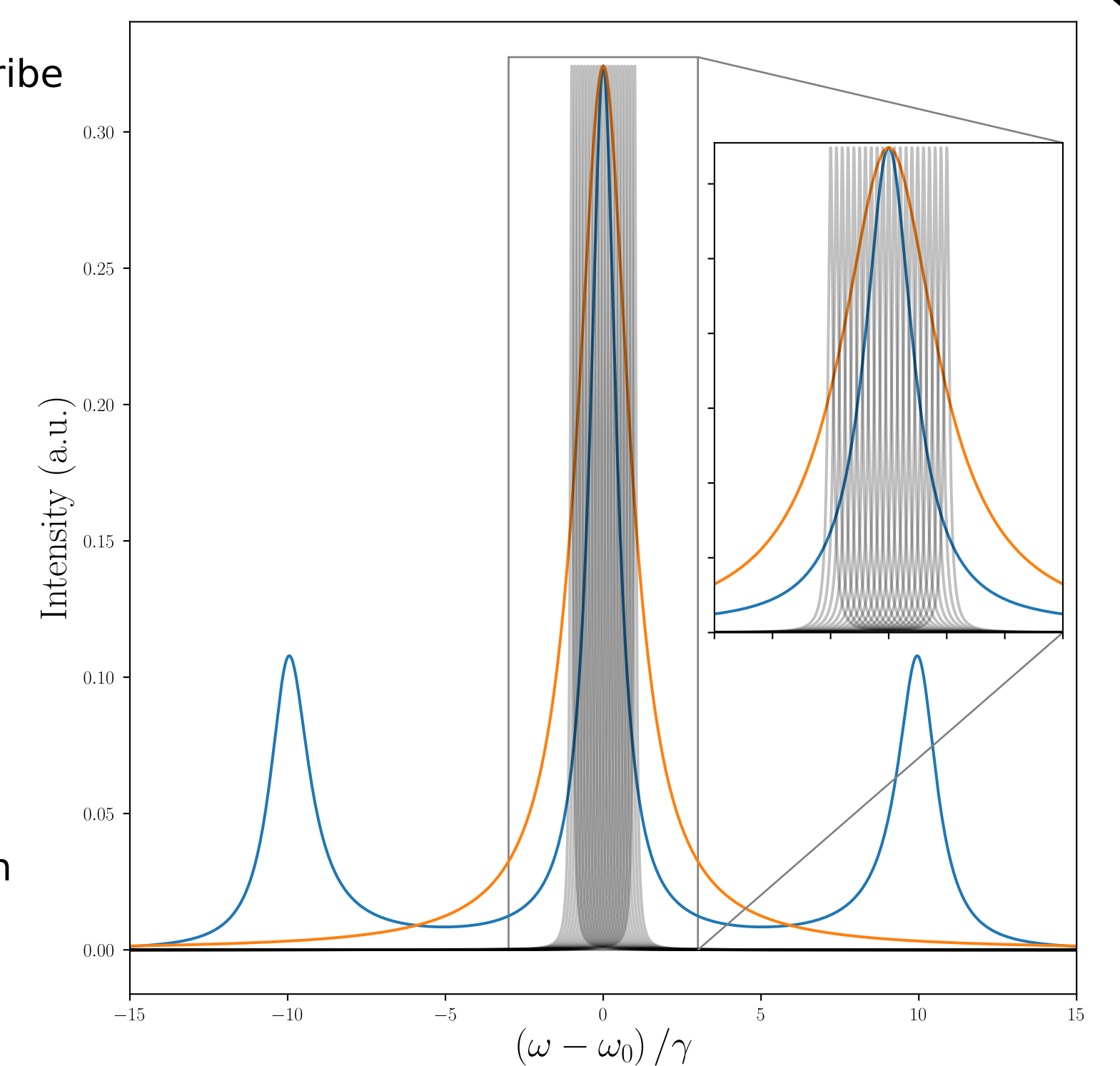
$$\frac{d\rho}{dt} = \frac{1}{i\hbar} [H, \rho] + \frac{1}{2} (2C\rho C^\dagger - C^\dagger C\rho - \rho C^\dagger C) + \frac{\kappa}{2} (2A\rho A^\dagger - A^\dagger A\rho - \rho A^\dagger A)$$

with where the two output channels, reflection and transmission, respectively, are described by "jump" operators

$$C = \sqrt{\gamma}\sigma_- + \sqrt{\kappa}A \quad A = \sum_{j=-N}^N a_j$$

where  $\Delta_j$  and  $a_j (a_j^\dagger)$  are the resonance frequency and photon annihilation (creation) operator for cavity mode  $j$ ,  $\kappa$  is the cavity mode decay rate (half-width). The central frequency of the cavity is  $\Delta_0$  and the filter has an effective bandwidth  $\Gamma = \Delta_N - \Delta_{-N}$ , with  $2N + 1$  evenly spaced modes. We investigate the filtered photon correlations of the different transitions by calculating the second-order correlation function of the filter cavity (in the steady state limit)

$$g^{(2)}(\tau) = \frac{\langle A^\dagger(0)A^\dagger(\tau)A(\tau)A(0) \rangle}{\langle A^\dagger A \rangle_{ss}^2}$$

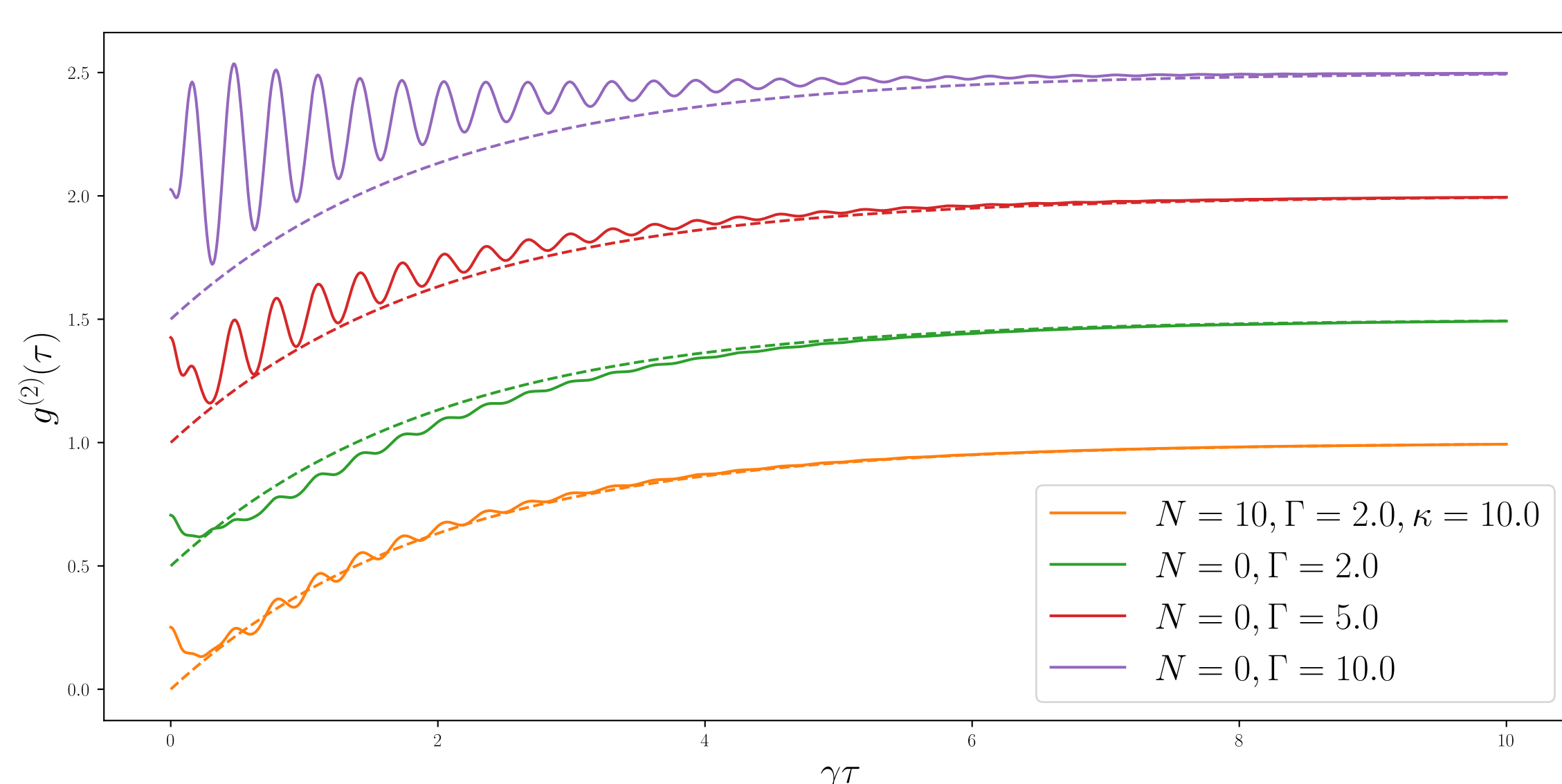
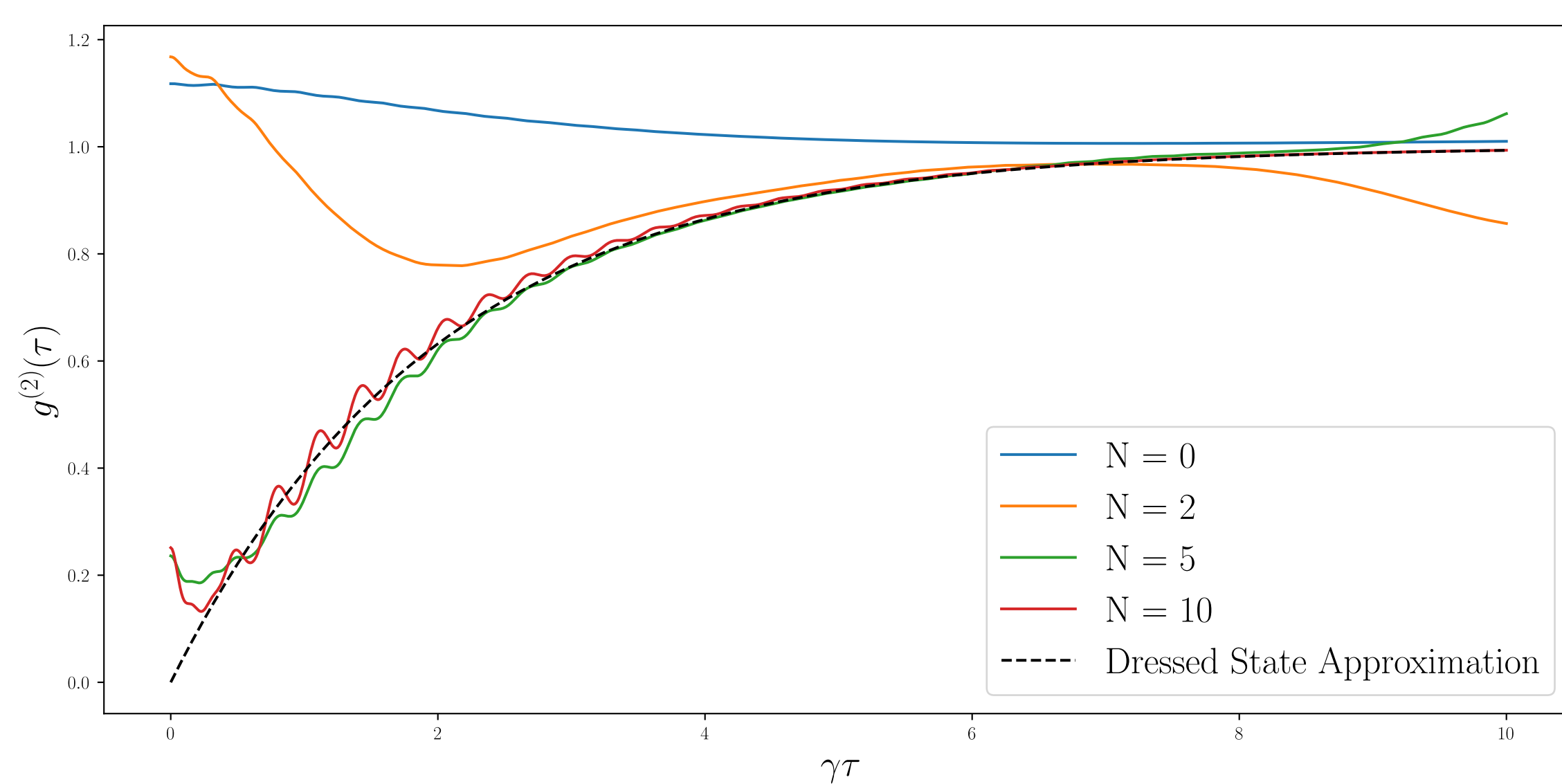


Our effective "box" filter (grey) is composed of  $2N + 1$  equally spaced Lorentzian filter modes. It has a much finer resolution compared to the single mode (orange)

## Single Mode Comparison

For the filter cavity to capture all of the atomic dynamics, the filter linewidth must be large enough to capture the oscillations, i.e.,  $\Omega$ . If we increase the number of cavity modes, however, the filter's resolution gets better.

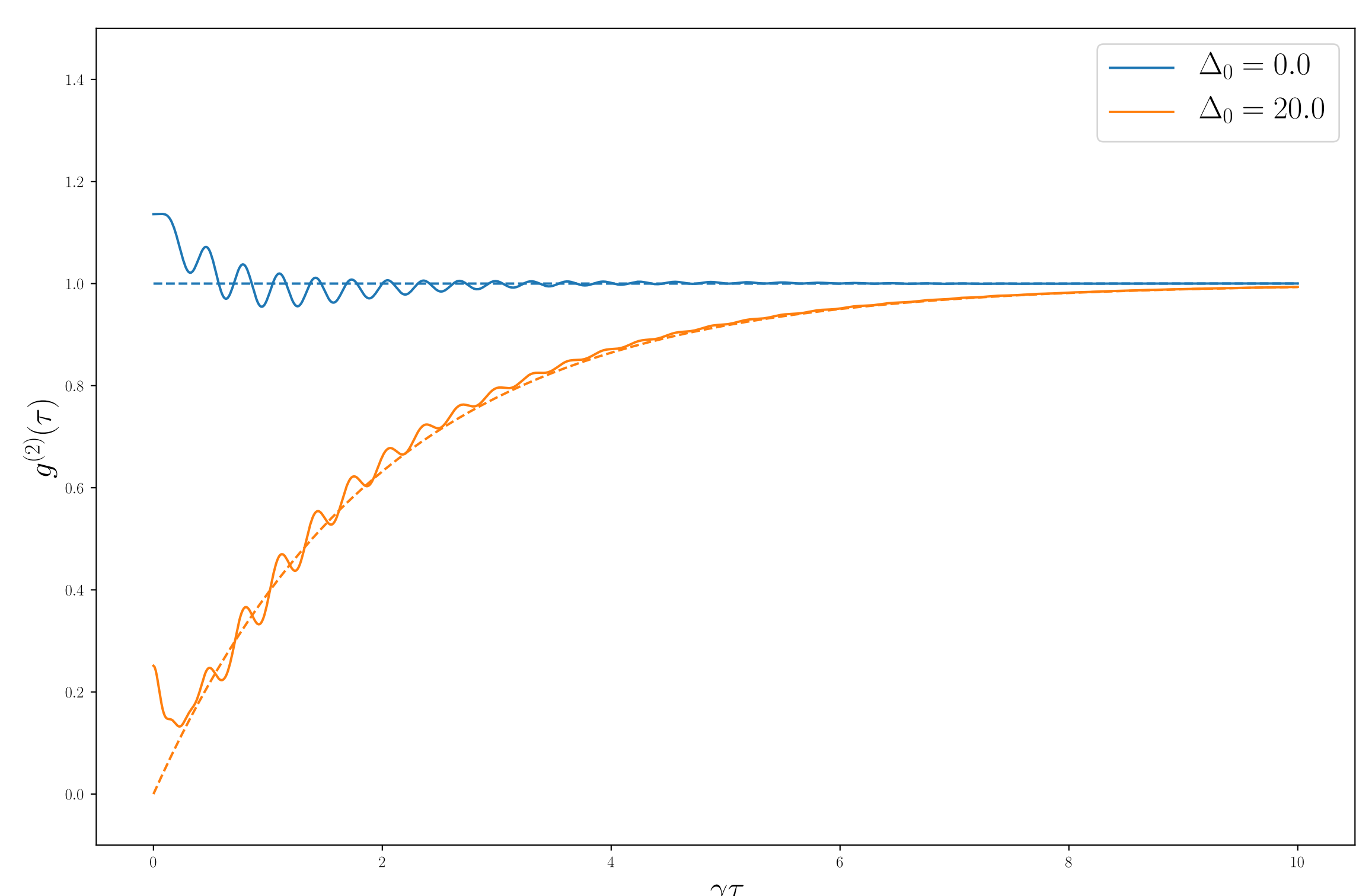
As the bandwidth of the filter increases, the probability of photons from transitions other than the target transition increases, allowing for unwanted contamination. In the bottom figure, we see the oscillation increasing as the overall trend moves away from the dressed state approximation with an increasing bandwidth.



$\Omega = 20.0, \Delta_0 = 20.0, \kappa = 0.1$  (top figure  $\Gamma = 2.0$ ). The dashed lines are correlations in the dressed state approximation.

## Dressed State Transitions

As long as the peaks in the power spectrum are well isolated in frequency space then the filter is able to resolve only the target transition, reproducing the expected behaviour from the dressed state approximation. Note that in the dressed state approximation, we have transformed into a frame rotating at the drive frequency, hence the lack of oscillations.



$\Omega = 20.0, \Delta_0 = 20.0, \kappa = 0.1, \Gamma = 2.0$ .

## References

- [1] Peng, Z., Yang, G., Wu, Q. & Li, G. *Phys. Rev. A* **99**, 033819 (2019).
- [2] Aspect, A., Roger, G., Reynaud, S., Dalibard, J. & Cohen-Tannoudji, C. *Phys. Rev. Lett.* **45**, 617-620 (1980).
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- [4] S. Gasparinetti, J. C. Besse, M. Pechal, R. Buijs, C. Eichler, H. J. Carmichael, and A. Wallraff, *Phys. Rev. A* **100**, 033802-1-8 (2019).