Filtered Photon Correlations of Fluorescence From a Driven Three-Level Atom

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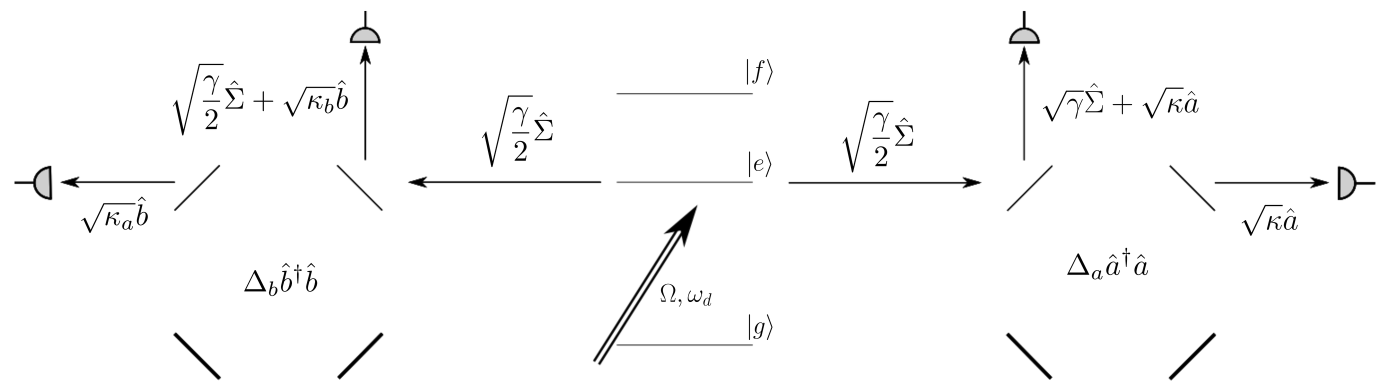
Introduction

In this work we develop a theoretical approach to investigate the nature of the fluorescence emitted by a driven three-level atom as studied by Gasparinetti et. al [1,2]. Using open systems theory to model a photon counting experiment, we employ frequency filtering techniques to isolate different peaks of the fluorescence spectrum and explore the second-order photon correlations.
The fluorescence is split and directed into two separate scanning interferometers (cavities) modelled as a cascaded system [3] with the Hamiltonian ($\hbar = 1$)



$$\hat{H} = \hat{H}_A + \Delta_a \hat{a}^\dagger \hat{a} + \Delta_b \hat{b}^\dagger \hat{b} + \frac{i}{2}\sqrt{\frac{\gamma\kappa_a}{2}}\left(\hat{\Sigma}^\dagger \hat{a} - \hat{\Sigma}\hat{a}^\dagger\right) + \frac{i}{2}\sqrt{\frac{\gamma\kappa_b}{2}}\left(\hat{\Sigma}^\dagger \hat{b} - \hat{\Sigma}\hat{b}^\dagger\right),$$

where Δ_a (Δ_b) is the cavity resonance frequency detuning from the drive frequency, κ_a (κ_b) is the full linewidth of cavity a (b), with respective photon annihilation and creation operators $\hat{a}$ ($\hat{b}$) and $\hat{a}^\dagger$ ($\hat{b}^\dagger$), γ is the atom decay rate and

$$\hat{\Sigma} = |g\rangle\langle e| + \xi\, |e\rangle\langle f|$$

is the atom lowering operator, with ξ the ratio of dipole moments for the two dipole transitions, $|g\rangle \leftrightarrow |e\rangle$ and $|e\rangle \leftrightarrow |f\rangle$; the atom has ground state $|g\rangle$, excited state $|f\rangle$ and intermediate state $|e\rangle$, with respective eigen-frequencies, ω_g, ω_f and ω_e. The Hamiltonian for the driven atom is

$$\hat{H}_A = -\left(\frac{\alpha}{2} + \delta\right)|e\rangle\langle e| - 2\delta\,|f\rangle\langle f| + \frac{\Omega}{2}\left(\hat{\Sigma} + \hat{\Sigma}^\dagger\right),$$

where Ω is the driving field strength (Rabi frequency), $2\delta = 2\omega_d - \omega_{fg}$ is the detuning of the drive frequency from the two-photon transition and $\alpha = \omega_{fe} - \omega_{eg}$, where $\hbar\omega_{ij} = E_i - E_j$.
Master equation

$$\frac{\mathrm{d}}{\mathrm{d}t}\hat{\rho} = \frac{1}{i\hbar}[\hat{H}, \hat{\rho}] + \frac{1}{2}\Lambda(\hat{J}_A)\hat{\rho} + \frac{1}{2}\Lambda(\hat{J}_B)\hat{\rho} + \frac{\kappa_a}{2}\Lambda(\hat{a})\hat{\rho} + \frac{\kappa_b}{2}\Lambda(\hat{b})\hat{\rho},$$

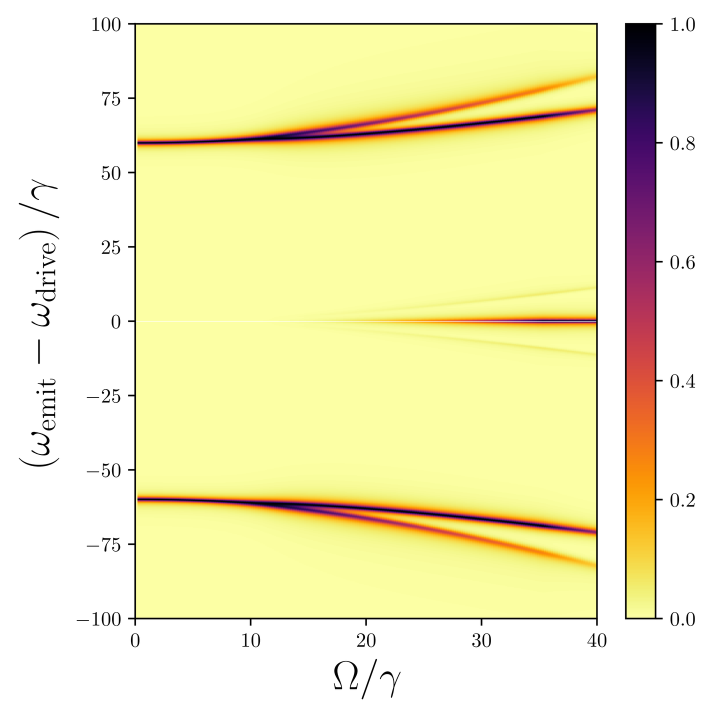
with Lindblad superoperator $\Lambda(\hat{A})\bullet = 2\hat{A}\bullet\hat{A}^\dagger - \hat{A}^\dagger\hat{A}\bullet - \bullet\hat{A}^\dagger\hat{A}$, and cascade decay operators $\hat{J}_a$ ($\hat{J}_b$) $= \sqrt{\gamma/2}\hat{\Sigma} + \sqrt{\kappa_a}\hat{a}$ $\left(\sqrt{\gamma/2}\hat{\Sigma} + \sqrt{\kappa_b}\hat{b}\right)$.

Unfiltered Two-Photon Resonance Fluorescence

The power spectrum is the Fourier transform of the first-order correlation function:

$$S(\omega) = \lim_{t\to\infty}\frac{1}{2\pi}\int_{-\infty}^{\infty}\mathrm{d}\tau\, e^{i\omega\tau}\langle\hat{\Sigma}^\dagger(t)\hat{\Sigma}(t+\tau)\rangle.$$

As the drive strength increases, we see dressed states appear [**Far Right**]. More transitions are available for the atom to decay from the state $|f\rangle$ to state $|g\rangle$, giving rise to the spectral splitting [**Right**].
The parameters are $(\alpha/\gamma, \delta/\gamma, \xi) = (-120.0, 0.0, 1.0)$.



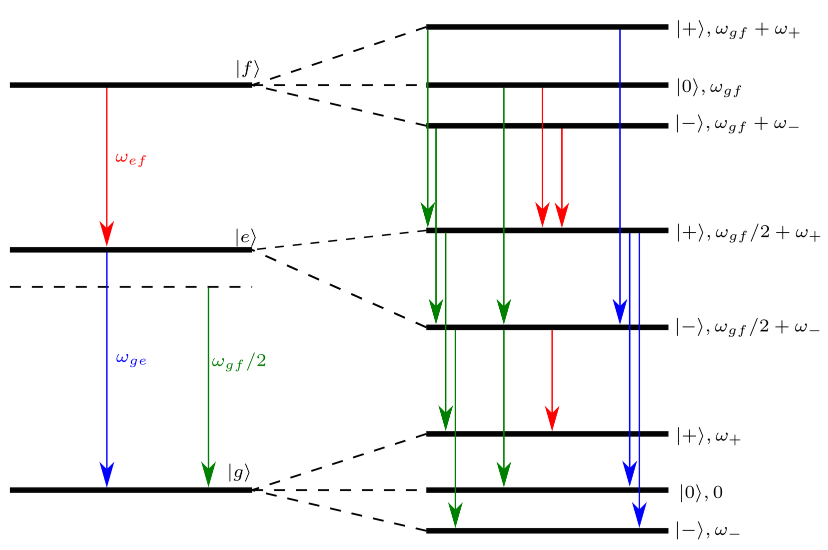
Filtered Photon-Photon Correlations

Tuning the ring cavities to specific transitions, we investigate the nature of the fluorescence from the photon auto-correlation of cavity a

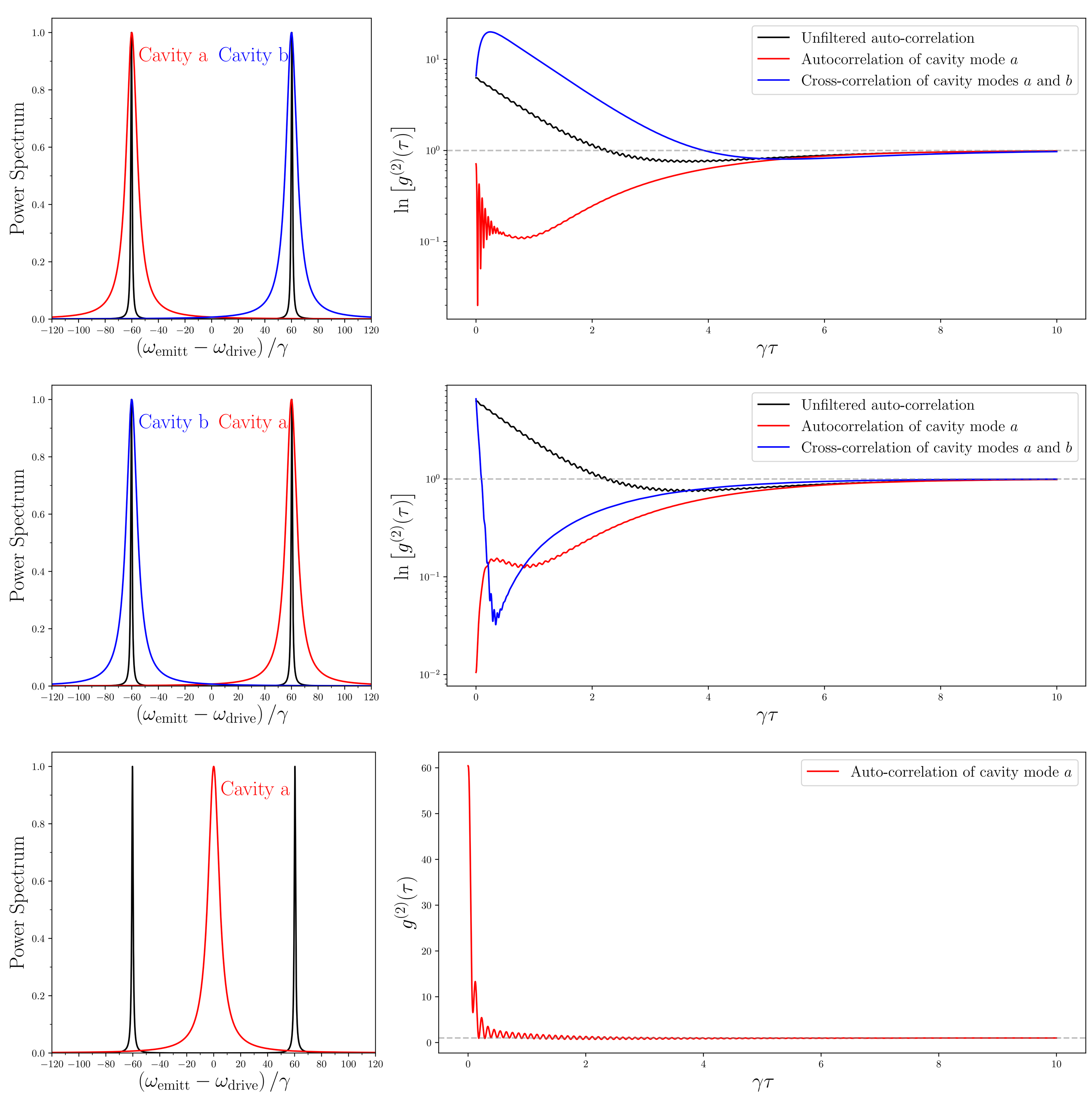
$$g^{(2)}_{\mathrm{Auto}}(\tau) = \lim_{t\to\infty}\frac{\langle\hat{a}^\dagger(t)\hat{a}^\dagger(t+\tau)\hat{a}(t+\tau)\hat{a}(t)\rangle}{\langle\hat{a}^\dagger(t)\hat{a}(t)\rangle\langle\hat{a}^\dagger(t+\tau)\hat{a}(t+\tau)\rangle},$$

and the conditional cross correlation where, given an emission from cavity a, we ask what is the likelihood of detecting an emission from cavity b after a delay τ

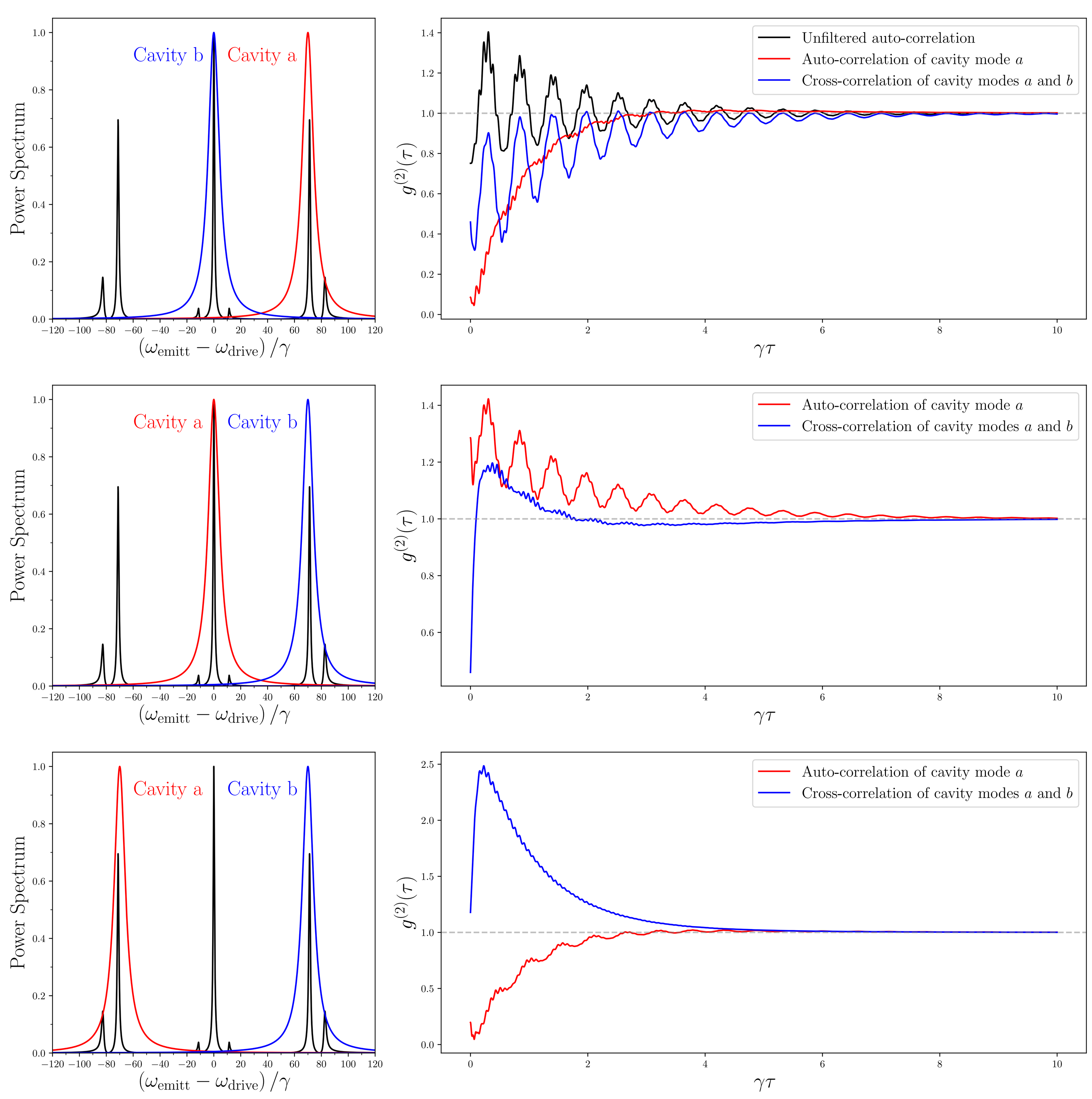
$$g^{(2)}_{\mathrm{Cross}}(\tau) = \lim_{t\to\infty}\frac{\langle\hat{a}^\dagger(t)\hat{b}^\dagger(t+\tau)\hat{b}(t+\tau)\hat{a}(t)\rangle}{\langle\hat{a}^\dagger(t)\hat{a}(t)\rangle\langle\hat{b}^\dagger(t+\tau)\hat{b}(t+\tau)\rangle}.$$



Weak Driving ($\Omega/\gamma = 5.0$)



Strong Driving ($\Omega/\gamma = 40.0$)



References

[1] Gasparinetti, S., Pechal, M., Besse, J.C., Mondal, M., Eichler, C. and Wallraff, A. "Correlations and Entanglement of Microwave Photons Emitted in a Cascade Decay". *Physical Review Letters*, 119(14), p.140504, 2017.
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[3] Charmichael, H.J. "Quantum Trajectory Theory for Cascaded Open Systems". *Physical Review Letters*, 70(15), p.2273, 1993.

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