Two-Photon Dressed States and Fluorescence Spectrum of a Driven Three-Level Atom

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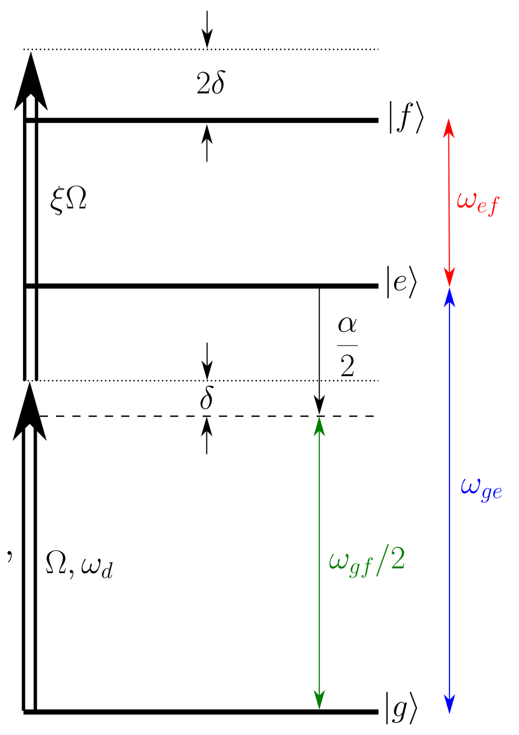
Introduction

We investigate the fluorescence spectrum of a three-level atom driven at two-photon resonance. We show that at high drives the spectrum displays a central triplet and two side-peak doublets. We also compute the second-order photon correlation function.

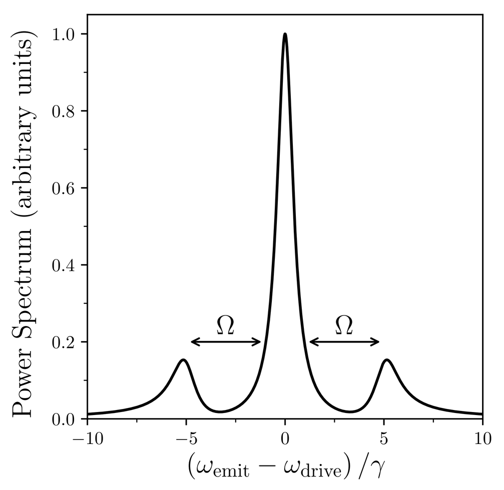
We consider a three-level ladder-type model with a ground state $|g\rangle$, final state $|f\rangle$, and intermediary state $|e\rangle$, respective frequencies ω_g, ω_f, and ω_e. For an external drive to be resonant with the $|f\rangle$ state, two photons of frequency $\omega_{gf}/2 = (\omega_f - \omega_g)/2$ must be absorbed. The Hamiltonian in matrix form in an interaction picture is ($\hbar = 1$) [1,2]



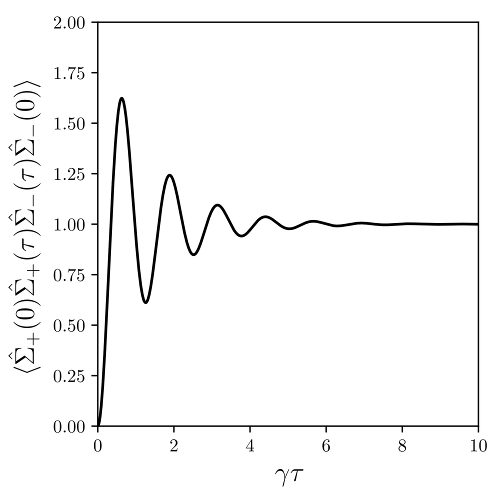
$$\hat{H} = \left(|g\rangle, \, |e\rangle, \, |f\rangle\right) \begin{pmatrix} 0 & \frac{\Omega}{2} & 0 \\ \frac{\Omega}{2} & -\frac{\alpha}{2} - \delta & \xi\frac{\Omega}{2} \\ 0 & \xi\frac{\Omega}{2} & -2\delta \end{pmatrix} \begin{pmatrix} \langle g| \\ \langle e| \\ \langle f| \end{pmatrix},$$

where Ω is the drive Rabi frequency, ξ is the ratio of dipole moments for the two dipole transitions, $\delta = \omega_d - \omega_{gf}/2$ is the detuning of the drive frequency from the two-photon transition, and $\alpha = \omega_{ef} - \omega_{ge}$ $(\omega_{ij} = \omega_j - \omega_i)$.

Single-Photon Resonance



$(\Omega/\gamma, \delta/\gamma, \alpha/\gamma, \xi) = (10.0, 60.0, -120.0, 1.0)$.



$(\Omega/\gamma, \delta/\gamma, \alpha/\gamma, \xi) = (10.0, 60.0, -120.0, 1.0)$.

If the drive frequency is resonant with the $|g\rangle \leftrightarrow |e\rangle$ transition ($\delta = -\frac{\alpha}{2}$) then the system effectively becomes a two level system, displaying the familiar Mollow triplet spectrum[3] and anti-bunching in the second order photon correlation.

Two-Photon Resonance: Dressed States

Considering the case where the drive is resonant with the two-photon transition ($\delta = 0$), by diagonalizing the Hamiltonian we find three dressed states:

$$|0\rangle = \frac{1}{\sqrt{1 + \xi^2}} \begin{pmatrix} -\xi \\ 0 \\ 1 \end{pmatrix},$$

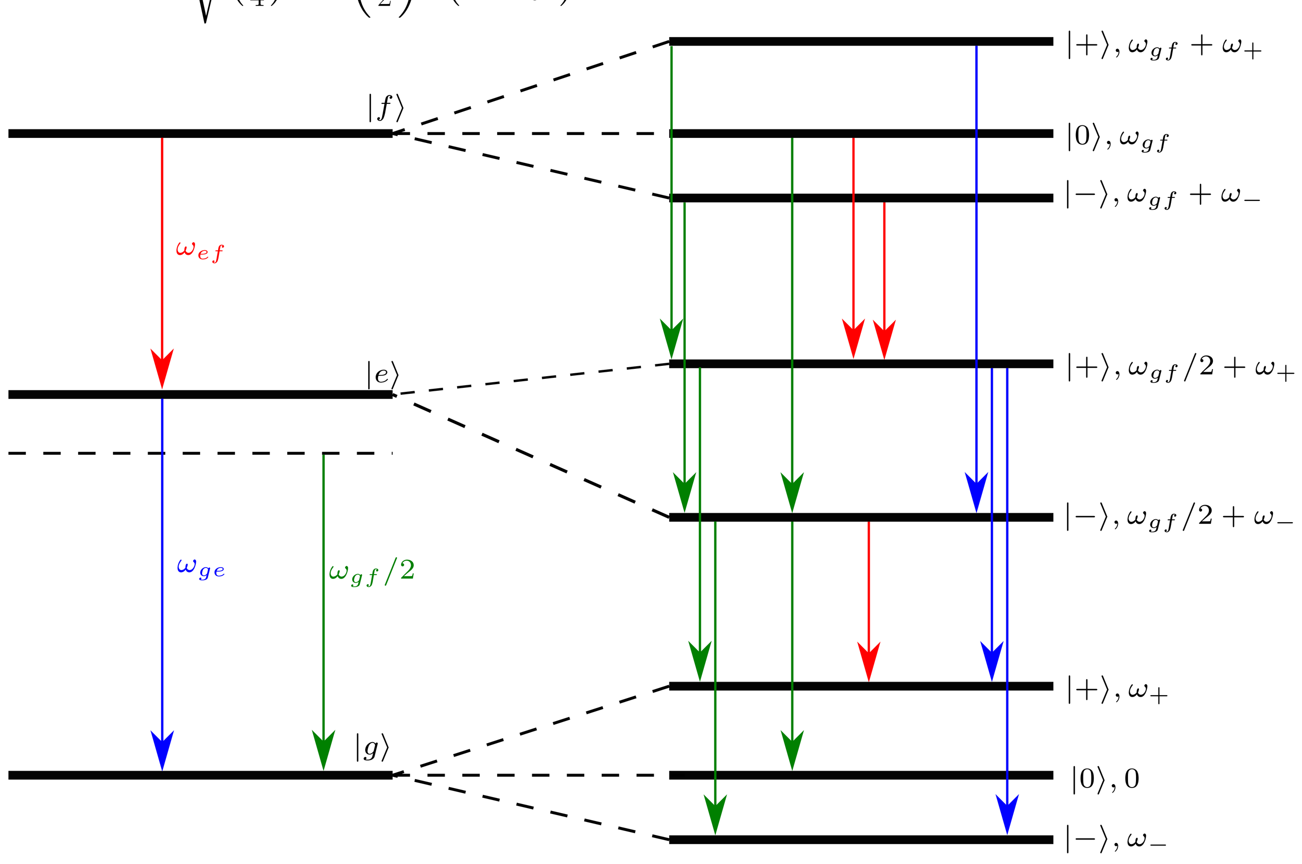
and

$$|\pm\rangle = \frac{1}{\sqrt{4\omega_\pm^2 + \Omega^2\left(1 + \xi^2\right)}} \begin{pmatrix} \Omega \\ 2\omega_\pm \\ \xi\Omega \end{pmatrix},$$

with eigen-frequencies

$$\omega_0 = 0 \text{ and } \omega_\pm = -\frac{\alpha}{4} \pm \tilde{\Omega},$$

where $\tilde{\Omega} = \sqrt{\left(\frac{\alpha}{4}\right)^2 + \left(\frac{\Omega}{2}\right)^2\left(1 + \xi^2\right)}$.



Two-Photon Resonance: Spectrum

We solve for the Power Spectrum using the Lindblad master equation

$$\frac{\mathrm{d}}{\mathrm{d}t}\hat{\rho} = \frac{1}{i\hbar}[\hat{H}, \hat{\rho}] + \frac{\gamma}{2}\left(2\hat{\Sigma}_-\hat{\rho}\hat{\Sigma}_+ - \hat{\Sigma}_+\hat{\Sigma}_-\hat{\rho} - \hat{\rho}\hat{\Sigma}_+\hat{\Sigma}_-\right),$$

where the decay operator is

$$\hat{\Sigma}_- = |g\rangle\langle e| + \xi\,|e\rangle\langle f|.$$

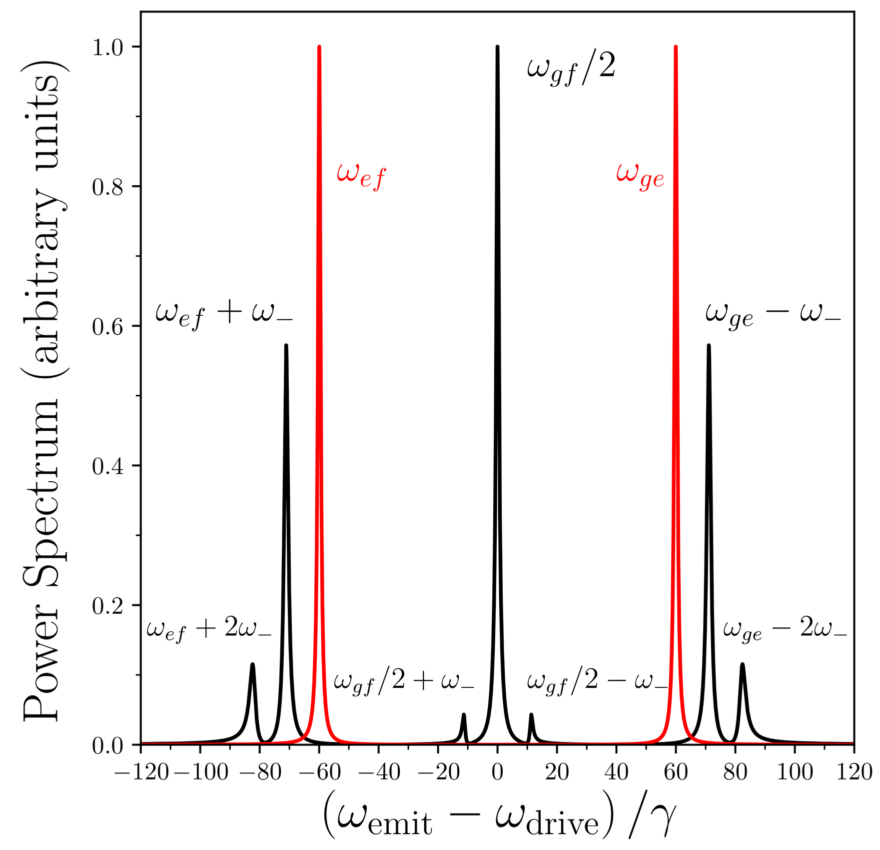
The spectrum is the Fourier Transform of the first-order correlation function,

$$S(\omega) = \lim_{t\to\infty} \frac{1}{2\pi} \int_{-\infty}^{\infty} \mathrm{d}\tau e^{i\omega\tau} \langle \hat{\Sigma}_+(t)\hat{\Sigma}_-(t+\tau)\rangle.$$

For the case of a low driving strength [**Right(red)**] we see the appearance of two distinct peaks corresponding to the cascade decay from $|f\rangle$ to $|e\rangle$ and then $|e\rangle$ to $|g\rangle$, with respective frequencies ω_{ef} and ω_{ge}.

As the drive strength increases, a central peak appears and splits into three and the two side-bands split into doublets [**Right(black)** and **Below left**].

We also see the splitting as the drive is brought closer to two-photon resonance [**Below right**].

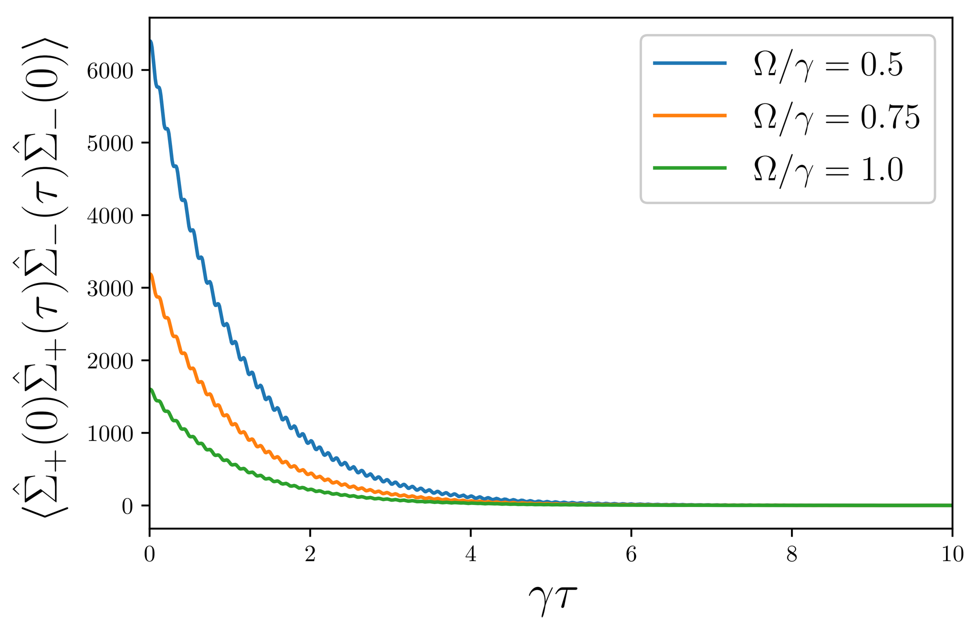


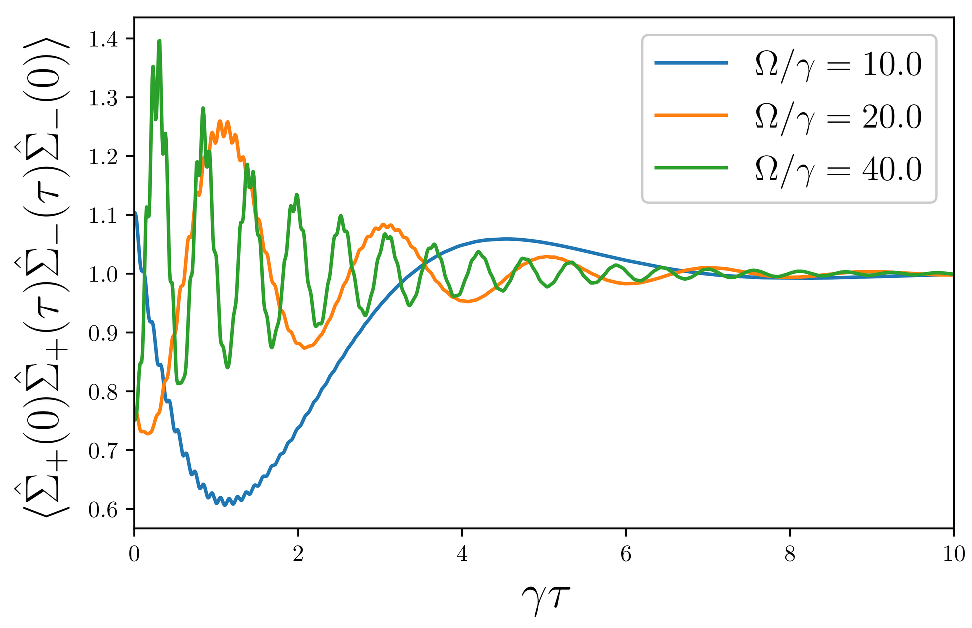
Normalised spectrum for $\Omega/\gamma = 40.0$ (1.0) in black (red). For both plots $(\delta/\gamma, \alpha/\gamma, \xi) = (0.0, -120.0, 1.0)$.

Spectrum splitting as a function of drive strength Ω. Parameters are $(\delta/\gamma, \alpha/\gamma, \xi) = (0.0, -120.0, 1.0)$.

Spectrum splitting as a function of detuning δ. Parameters are $(\Omega/\gamma, \alpha/\gamma, \xi) = (40.0, -120.0, 1.0)$.

Photon Correlation





We calculate the second order correlation function

$$g^{(2)}(\tau) = \lim_{t\to\infty} \frac{\langle \hat{\Sigma}_+(t)\hat{\Sigma}_+(t+\tau)\hat{\Sigma}_-(t+\tau)\hat{\Sigma}_-(t)\rangle}{\langle \hat{\Sigma}_+(t+\tau)\hat{\Sigma}_-(t+\tau)\rangle\langle \hat{\Sigma}_+(t)\hat{\Sigma}_-(t)\rangle}.$$

As the drive strength decreases, $g^{(2)}(0) \to \infty$ indicating that the system acts as a two-photon source [**Above left**]. As the drive strength increases, Rabi oscillations occur in the correlation and $g^{(2)}(0) < 1$ indicating anti-bunched light [**Above right**].

References

[1] Gasparinetti, S., Pechal, M., Besse, J.C., Mondal, M., Eichler, C. and Wallraff, A. "Correlations and entanglement of microwave photons emitted in a cascade decay". *Physical Review Letters*, 119(14), p.140504, 2017.

[2] Gasparinetti, S., Buijs, R. D., Wallraff, A., et. al. "Two-photon resonance fluorescence of a weakly nonlinear artificial atom", *unpublished*.

[3] Mollow, B. R, "Power Spectrum of Light Scattered by Two-Level Systems", *Physical Review*, 188(5):1969-1975, 1969.

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