Phase Resetting in the Yamada Model of a *Q*-Switching Laser

J. Ngaha[1,2], N. G. R. Broderick[2,3], and B. Krauskopf[1,2]

[1] Department of Mathematics, The University of Auckland, Private Bag 92019, Auckland, New Zealand
[2] The Dodd-Walls Centre for Photonic and Quantum Technologies, New Zealand
[3] Department of Physics, The University of Auckland, Private Bag 92019, Auckland, New Zealand

In this work we investigate the phase resetting of a self pulsing laser, also known as a *Q-switching* laser, with a saturable absorber, consisting of a gain and an absorber section. We study a specific model, due to Yamada [1, 2], consisting of a set of three coupled ordinary differential equations for the gain G, the absorption Q, and the intensity I.

We take a dynamical systems approach and employ recently developed methods [3] to address how the regular self-pulsating behaviour of a Q-switching laser is affected by external perturbations. In Fig. 1 we consider an intensity perturbation, given by $\vec{d}_{\rm p} = (0, 0, 1)$ and amplitude $A_{\rm p} > 0$. Figure 1a depicts the time-series of the intensity of an unperturbed periodic orbit, along with an example of a perturbed orbit. The perturbation is applied at $t = 0$, initially advancing the oscillations compared to the unperturbed orbit. The perturbed orbit eventually returns to the same pulsing behaviour at $t \sim 300$, though with a slight delay of $\Delta t \approx 4.5$.

Figure 1(b) depicts phase transition curves (PTCs) for four different perturbation amplitudes. This represents the relationship between the phase of the periodic orbit where the perturbation is applied, $\theta_{\rm old}$, and the shifted phase, $\theta_{\rm new}$, where each phase specifies a point on the periodic orbit. If no perturbation is applied, the two phases are the same and thus the PTC is the identity (dashed line in Fig. 1b). For weaker perturbations, we see qualitatively similar behaviour to the unperturbed case, as the new phase $\theta_{\rm new}$ changes over 2π (as does $\theta_{\rm old}$). For much larger perturbations, we see completely different behaviour. As the perturbation is applied along the original periodic orbit, there is no longer an increase of 2π in $\theta_{\rm new}$, as shown for $A_{\rm p} = 25$ and 30 in Fig. 1b.

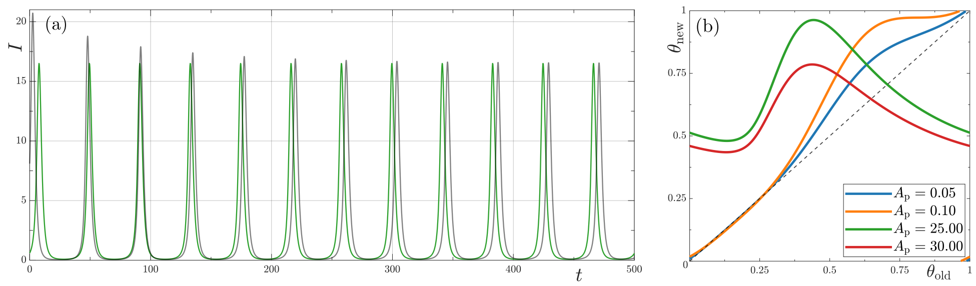

Figure 1: (a) Time series of the unperturbed pulsations (green). The perturbed response (black) returns back to the original pulsations at $t \sim 300$, with a delay. The perturbation is applied in the positive I-direction, $\vec{d} = (0, 0, 1)$, with amplitude $A_{\rm p} = 7.5$ at $t = 0$. (b) PTCs for four different perturbation amplitudes as shown. The PTCs for the two weaker perturbations are close to the identity (black, dashed), while those for the two larger perturbations, are qualitatively different.

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[3] P. Langfield, B. Krauskopf, and H. M. Osinga. “A continuation approach to computing phase resetting curves” in *Advances in Dynamics, Optimization and Computation”*, Vol. 304 of *SSDC* (Springer International Publishing, 2020), pp.3-30.